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## On the Elementary Explanation of Diffusion Phenomena in Gases


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THE mathematical theory of transport phenomena in gases, including the phenomena of diffusion, was put on a satisfactory foundation by the development of Enskog's[1] method for the solution of Boltzmann's equation. A discussion of the method and of most of the calculations made with it is given in the treatise published several years ago by Chapman and Cowling.[2] Several chapters are devoted to the exposition of the formulas and numerical results obtained from the calculations, and to comparisons with experimental data. The book seems well suited to meet the needs of research workers dealing with transport phenomena.

Despite the satisfactory state of the mathematical theory and of its exposition, there remains a need for simple physical interpretations of these processes. Many, and probably most, scientists are inclined to feel that while the precise mathematical theory of a phenomenon has its place in the final working out of any particular problem, a simple conception of the physical nature of the processes involved is of much greater usefulness in developing new ideas, choosing methods, and designing apparatus. For teaching purposes, elementary explanations are obviously indispensable.

In the case of transport phenomena, the present standard answer to the call for a physical explanation is given by arguments based on the concept of mean free path. As applied to viscosity and thermal conductivity, these arguments succeed in accounting for the main experimental facts in quite a satisfactory fashion. For diffusion phenomena, however, treatments based on the concept of mean free path lead to results that are in distinct qualitative disagreement with experiment, and also, of course, with the results of the detailed mathematical theory. The fact that such arguments do not provide a correct physical approach to diffusion problems has been particularly emphasized by Chapman.[3]

Elementary treatments based on the idea of the mean free path continue, nevertheless, to enjoy widespread use, and in most presentations[4] no indication whatever is given that the results are less satisfactory in the case of diffusion than they are for the other transport phenomena. The need for a better elementary discussion of diffusion problems has in recent years been felt mainly through the desire for a physical explanation of thermal diffusion. Repeated attempts at an explanation in terms of mean free path have been made, the most nearly successful being that of Fürth.[5]

The demand for a simple physical explanation

[1] D. Enskog, *Dissertation* (Upsala, 1917).
[2] S. Chapman and T. G. Cowling, *The mathematical theory of nonuniform gases* (Cambridge Univ. Press, 1939).
[3] S. Chapman, *Phil. Mag.* **5**, 630 (1928).
[4] Prominent exceptions are Kennard's *Kinetic theory of gases*, Herzfeld's *Kinetische Theorie der Wärme*, and Jeans' *Dynamical theory of gases*. Jeans mentions the specific cases He$-$A and $H_2-CO_2$ to which we refer in SEC. 1.
[5] R. Fürth, *Proc. Roy. Soc.* **A179**, 461 (1942). A shortened and simplified form of the argument is given by R. N. Rai and D. S. Kothari, *Ind. J. Physics* **17**, 103 (1943).

of thermal diffusion receives a satisfactory answer through a method devised by Frankel[6] and stated by him only very briefly. Examination at somewhat greater length shows that it certainly gives an appropriate physical approach to the problem, and provides results as correct and complete in every respect as could reasonably be expected of an elementary physical discussion. By contrast, Fürth's treatment, despite its skillful presentation and partial success, would seem to be useful mainly in showing the inappropriateness of the concept of mean free path as an approach to this problem.

Frankel's method has been used by Cacciapuoti[7] in a detailed treatment of thermal diffusion for the case of rigid spherical molecules. In the present paper we wish not only to give a more general discussion of the application to thermal diffusion, but also to show that Frankel's approach is the really suitable one for the elementary explanation of ordinary diffusion as well.

The idea used by Frankel is essentially an improved form of a suggestion made long ago by Stefan,[8] who used it to give a detailed and quite satisfactory treatment of diffusion for the case of rigid spherical molecules. This calculation was outstandingly good compared to other approximate work on the problem, but for some reason was rather generally ignored. Stefan's basic physical hypothesis seems to have had the approval of Maxwell, and to have been used by him in obtaining certain results.[9] Maxwell did not, however, give any clear explanation of his way of using the idea.[10] It received mention in the subsequent literature only occasionally, and then usually only as a special assumption which might be taken into account as modifying the results obtained by the mean-free-path approach,[11] not as an independent basis for a direct physical approach to diffusion problems. The revival of this independent method had to wait until the rather recent work of Frankel.

We shall give a clear formulation of the Stefan-Frankel method and apply it to two fundamental problems in connection with which the traditional elementary treatments are decidedly unsatisfactory. In Sec. **1** we shall discuss in greater detail some of the difficulties of the mean-free-path treatment of diffusion. The general formulation of the method of Stefan and Frankel is given in Sec. **2**. In Sec. **3** we apply this method to the problem of ordinary diffusion in a binary mixture of gases, with particular regard to the question of the dependence of diffusion coefficient on concentration, which was given altogether incorrectly by the traditional elementary treatment. In Sec. **4** we apply the method to the problem of thermal diffusion.

## 1. CRITICISM OF THE MEAN-FREE-PATH METHOD

### General Nature of the Method

The elementary treatment of transport phenomena based on the concept of mean free path begins by considering all the molecules that cross a certain element of area per unit time. The transport of the quantity in question across this area is then calculated on the assumption that its value for each molecule is determined by the position at which the molecule suffered its last collision, the average coordinates of this position being determined by using the idea of mean free path.

For the cases of transport of momentum (viscosity) and of energy (thermal conductivity) there is no reason, either in principle or in the results obtained, for refusing to accept this as a thoroughly appropriate simple physical approach to these problems. In the case of diffusion (transport of molecular types themselves), however, the method is neither really plausible nor even tolerably successful. We shall return later to the discussion of its lack of plausibility, which provides arguments favoring the point of view of Stefan and Frankel. Its lack of success will be illustrated at once.

### Failure to Agree with Experiment

The most striking evidence of the failure of the traditional elementary method to give results

[6] S. Frankel, *Physical Rev.* **57**, 660 (1940).
[7] B. N. Cacciapuoti, *Nuovo Cimento* (9) **1**, 126 (1943).
[8] J. Stefan, *Wien. Sitzungsber.* **65**, 323 (1872).
[9] See J. C. Maxwell, *Scientific papers*, Vol. II, p. 345. This paper gives results and discussion only, not calculations, and seems to have been published a little before Stefan's.
[10] In an article in the *Encyclopedia Brittanica*, Maxwell used Stefan's approach in giving a *macroscopic* discussion of diffusion quite similar to Stefan's own (reference 9, p. 630).
[11] See J. Jeans, *Dynamical theory of gases* (Cambridge Univ. Press; ed. 4), p. 315.

agreeing with experimental facts is obtained by considering the dependence of the diffusion coefficient on concentration. If in a binary mixture only a small fraction of the molecules are of type 1, the result obtained for the diffusion coefficient is

$$D_{12}=A\bar{c}_1\lambda_1. \quad \text{(1 rare)} \qquad (1)$$

Here $\bar{c}_1$ is the mean speed and $\lambda_1$ the mean free path of molecules of type 1; the constant $A$ is given the value $\frac{1}{3}$ by the usual simple argument, but was given the value $\pi/8$ by Meyer[12] on the basis of a more pretentious calculation. The occurrence of $\bar{c}_1$ and $\lambda_1$ only corresponds to a physical picture in which a few molecules of type 1 diffuse through the preponderant type 2 gas. If, on the other hand, molecules of type 2 are rare and type 1 is preponderant, the result is

$$D_{12}=A\bar{c}_2\lambda_2. \quad \text{(2 rare)} \qquad (2)$$

For the general case, an argument given by Meyer[12] and repeated almost universally gives

$$D_{12}=A(\gamma_2\bar{c}_1\lambda_1+\gamma_1\bar{c}_2\lambda_2). \qquad (3)$$

Here $\gamma_1$ and $\gamma_2$ are the fractional numerical concentrations:

$$\gamma_i=n_i/n=n_i/(n_1+n_2), \quad \gamma_1+\gamma_2=1, \qquad (4)$$

$n_i$ being the number of molecules of type $i$ per unit volume. The dependence of $D_{12}$ on $\gamma_1$ and $\gamma_2$ is rather more complicated than is shown explicitly in Eq. (3), because $\lambda_1$ and $\lambda_2$ themselves depend on concentration.

Without going into the details of the dependence on concentration, we can see at once that for decidedly dissimilar molecules Eqs. (1) to (3) predict that the value of $D_{12}$ will change rapidly with changes of concentration. This is true especially if the two types of molecule differ greatly in mass. According to the usual ways of calculating mean free path, one finds that[13]

$$\frac{(\lambda_1)_{1\text{ rare}}}{(\lambda_2)_{2\text{ rare}}}=\left(\frac{m_2}{m_1}\right)^{\frac{1}{2}}. \qquad (5)$$

Since, of course,

$$\bar{c}_1/\bar{c}_2=(m_2/m_1)^{\frac{1}{2}}, \qquad (6)$$

we have, from Eqs. (1) and (2),

$$\frac{(D_{12})_{1\text{ rare}}}{(D_{12})_{2\text{ rare}}}=\frac{m_2}{m_1}. \qquad (7)$$

Thus, for instance, the value of $D_{12}$ for the diffusion of a small admixture of hydrogen through carbon dioxide should be *22 times* as large as its value for the case of a small admixture of carbon dioxide diffusing through hydrogen. For the case of helium and argon the corresponding ratio should be ten. *Such predictions are flatly contradicted by experiment.* For example, in both of the rather extreme cases mentioned the total change of $D_{12}$ with concentration is[14] not more than 15 percent.

This pronounced difficulty with the results of the mean-free-path approach can be alleviated somewhat by resort to considerations[15] based on "persistence of velocity," which effectively make the ratio of mean free paths different from that indicated in Eq. (5). The calculations required, however, are long and complicated, and for cases such as those mentioned there is no way of seeing without considerable labor that the correction would be just enough to make $D_{12}$ essentially independent of concentration. The partial success of such attempts at "patching up" the results cannot prevent the judgment that the basic approach used is not at all well adapted to the physical nature of the process. It can scarcely be doubted that the physicists of 70 years ago would have discarded the method of Meyer and given

[12] O. E. Meyer, *Kinetische Theorie der Gase* (Breslau, ed. 2, 1899), pp. 252–263.

[13] Both the mutual cross-sectional areas and the distribution of relative velocities of dissimilar molecules are independent of concentration. Thus the mean free time for the rare type of molecules is the same, no matter which type is rare. The mean free paths in question are thus proportional to the mean speeds, and Eq. (5) follows from Eq. (6). Both Fürth and Rai and Kothari (reference 5) calculate mean free paths incorrectly, taking all molecules except the one in question to be at rest. Perhaps the propensity to this oversimplification comes from the fact that most considerations of mean free path in the last decade have referred to neutron scattering.

[14] Chapman and Cowling, reference 2, p. 248. The data given do not extend to the limiting concentrations indicated in Eq. (7), but can be extrapolated roughly either graphically or by the use of the results of the Enskog-Chapman theory, which agree well with experiment.

[15] Jeans, reference 11, pp. 313–315, gives a brief indication of one such procedure. The results for the case of rigid spherical molecules are fairly good, but no other cases have been treated. The correction affects only the dependence of $D_{12}$ on concentration owing to difference in masses of the molecules, and only some very different and probably much more complicated procedure could have any bearing on dependences arising from other causes (see footnote 19).

that of Stefan a prominent place in the literature of the subject, if it had not been for the fact that reliable experimental evidence was lacking at that time.

### Lack of Physical Plausibility

The simple treatment of the problem of diffusion by means of the concept of mean free path actually lacks plausibility both as to method and as to results. First as to method: In diffusion the transport associated with the passage of a molecule through a given element of area depends only on the fact of its passage and on its permanent property of belonging to one or another species. The question of the locale of its last collision is, accordingly, much less pertinent than it is in the cases of the other transport phenomena, in which the property transported—momentum or energy—is strongly affected by that collision. The previous positions of the particle do, of course, provide statistical evidence as to its probable species, and this is the only evidence available. But instead of yielding a fairly decisive clue, as it does in the problems of viscosity and thermal conductivity, the last collision here serves in large part just to mark the point at which the trail becomes quite hard to retrace. The Meyer method contents itself with the evidence easily obtainable, and gets results that are too inaccurate to be really acceptable. It is physically correct to seek more evidence from considerations of "persistence of velocities," but the resulting task is tedious, unilluminating and not even completely feasible. We shall see that the Stefan-Frankel method succeeds in obtaining genuinely decisive evidence from consideration of single collisions. This is possible because in this method the effect of the collision on a dynamical quantity—the momentum—is studied.

The implausibility of the *result* of the simple mean-free-path treatment of diffusion is evident from the form of the result itself, as shown in Eq. (3), without any consideration of the underlying method. We may regard the process of diffusion as one in which each gas is expanding to fill the whole volume uniformly, but the expansion of each gas is hindered by collisions of its molecules *with molecules of the other type* (*or types*). Collisions between molecules of the same type affect only the fortunes of individuals, not the flow of the gas.[16] The diffusion coefficient which measures the rapidity of the process, should, accordingly, be capable of being calculated approximately in terms of quantities that are of the general nature of mean free paths but are to be evaluated by taking into account collisions of *unlike molecules* only. These quasi free paths might be denoted by $\lambda_i'$. Since Eq. (3) contains, instead, the ordinary mean free paths $\lambda_1$, $\lambda_2$, which depend on other quantities besides $\lambda_1'$, $\lambda_2'$ and may differ from the latter by large factors, it is obviously physically incorrect.

It is sometimes suggested[11] that this defect of Eq. (3) be corrected by simply replacing $\lambda_1$, $\lambda_2$ by $\lambda_1'$, $\lambda_2'$. Since

$$\lambda_1' = \gamma_2^{-1}(\lambda_1)_{1\ \text{rare}}, \quad \lambda_2' = \gamma_1^{-1}(\lambda_2)_{2\ \text{rare}}, \tag{8}$$

this gives a result independent of concentration. The obtaining of a qualitatively improved result by such irresponsible doctoring of a wrong formula can only be regarded as fortuitous; the only consistent procedure for improving Eq. (3) would be an elaborate one based on the general idea of "persistence of velocities." It is unfortunate that this meaningless trick has come to be rather commonly identified as Stefan's contribution to the theory, while his radically different, simple and physically correct method has been neglected.

## 2. THE STEFAN-FRANKEL METHOD

### Basic Idea of the Method

Stefan's way of approaching the problem of diffusion is just the opposite of the traditional elementary approach, which uses the concept of mean free path. Instead of starting from an assumed gradient of concentration and calculating from it the flux of each molecular species relative to the motion of the gas as a whole, we begin by assuming the relative flux of the different species and then proceed to calculate the corresponding gradient of concentration. We shall describe the procedure for a mixture of two components; this is the standard case both in the rigorous theory and in the various attempts at elementary explanation.

[16] The discussion in at least one otherwise excellent elementary textbook offers serious possibilities of confusion by failing to make this point clear; see Millikan, Roller and Watson, *Mechanics, molecular physics, heat, and sound* (Ginn, 1937), pp. 218–219.

In a homogeneous gas mixture of moderate density the number density $n_i$ of each molecular species is related to its partial pressure $p_i$ by the simple equation

$$p_i = n_i kT, \tag{9}$$

which is well known from the most elementary discussions of kinetic theory. Also, if $p$ is total pressure and $n$ is total number density,

$$p = nkT. \tag{10}$$

The partial pressure $p_i$ can then be expressed in terms of the fractional numerical concentration $\gamma_i$, defined in Eq. (4):

$$p_i = \gamma_i nkT = \gamma_i p. \tag{11}$$

If the mixture is not homogenous but has a small gradient of concentration, Eq. (11) will still apply, at least in excellent approximation, to each small part of the volume. The total pressure being supposed constant, we find by differentiating Eq. (11) that

$$(\partial/\partial x)p_i = nkT(\partial/\partial x)\gamma_i = p(\partial/\partial x)\gamma_i. \tag{12}$$

This relation between gradient of partial pressure and gradient of concentration holds both for constant temperature and for moderate temperature gradients.

The concentration gradient is thus determined if we can calculate the gradient of partial pressure. The latter quantity can readily be computed from suitable assumptions about the motions of the various species of molecule. Indeed, it is evident that the gradient of the partial pressure is equal to the time rate, per unit volume, of transfer of momentum from molecules of other types to molecules of the type in question.

## Transfer of Momentum in Molecular Encounters

*Encounter between two molecules.*—The transfer of momentum in an encounter between two molecules is most readily calculated by using the reference system of the center of mass. If $\mathbf{V}$ is the relative velocity, so that

$$\mathbf{V} = \mathbf{c}_1 - \mathbf{c}_2, \tag{13}$$

then before the encounter the velocity of the first molecule in this system is $m_2\mathbf{V}/(m_1+m_2)$ and its momentum is

$$m_1 m_2 \mathbf{V}/(m_1+m_2) \equiv \mu\mathbf{V}; \tag{14}$$

here $m_1$, $m_2$ are the masses of the molecules, and $\mu$ is the "reduced mass." The velocity of the second molecule before the encounter is then $-m_1\mathbf{V}/(m_1+m_2)$, and its momentum is $-\mu\mathbf{V}$. In the reference system chosen both paths have the same angle of deflection $\psi$, and the momentums after the encounter are still opposite in direction and each of magnitude $\mu V$. The momentum transferred to the first molecule consists of a component

$$\delta\mathbf{p}_{\parallel} = \mu\mathbf{V}\cos\psi - \mu\mathbf{V} = -\mu\mathbf{V}(1-\cos\psi) \tag{15}$$

in a direction parallel to $\mathbf{V}$ and a component

$$\delta\mathbf{p}_{\perp} = \mu V \sin\psi \mathbf{b}_1 \tag{16}$$

in a perpendicular direction; here $\mathbf{b}_1$ is a unit vector normal to $\mathbf{V}$. Since all directions of $\mathbf{b}_1$ normal to $\mathbf{V}$ are equally probable, the sum of the contributions given by Eq. (16) is zero when we consider the effect of many encounters.

For given relative speed $V$ of two spherically symmetrical molecules of given types the angle of deflection $\psi$ depends only on the impact parameter $b$; this is the distance of closest approach of the centers of mass of the two molecules that would result if they were to pursue their original paths without any deflection. If the molecules are not spherically symmetrical, $\psi$ depends also on their orientations. Since, however, all orientations are equally probable, this merely makes necessary an averaging process; we shall not concern ourselves with this, and shall treat simply the case of spherical molecules.

*Interaction between sets or beams of molecules.*—We now consider a small fraction of the molecules of type 1, having velocities essentially equal to $\mathbf{c}_1$ and number density $dn_1$, together with a similar fraction of type 2, with velocity $\mathbf{c}_2$ and number density $dn_2$. The number of collisions between members of the two groups per unit volume and unit time with impact parameter between $b$ and $b+db$ is

$$dn_1 dn_2 V \cdot 2\pi b db. \tag{17}$$

Then, using Eq. (15), we find that the momentum transferred to members of the first group from members of the second is, per unit volume and unit time,

$$-dn_1 dn_2 \mu\mathbf{V} V \int_0^\infty 2\pi b db(1-\cos\psi) \tag{18}$$

The quantity

$$\sigma(V) \equiv \int_0^\infty 2\pi b\mathrm{d}b(1-\cos\psi), \tag{19}$$

can be called the cross section for transfer of momentum. Its special suitability for our purpose comes from the particular factor $1-\cos\psi$ in the integrand. In the study of scattering and absorption of fast particles the quantity of this kind most generally used is the "total cross section,"

$$\sigma_{\mathrm{tot}} = \int_0^{b_{\max}} 2\pi b\mathrm{d}b = \pi b_{\max}^2, \tag{20}$$

where $b_{\max}$ is the largest value of the impact parameter that gives a deflection regarded as appreciable. In the Enskog-Chapman theory of transport phenomena the quantity $\sigma(V)$ defined by Eq. (19) is the cross section most important for diffusion, while for the phenomena of viscosity and thermal conductivity the cross section

$$\sigma_2(V) = \int_0^\infty 2\pi b\mathrm{d}b(1-\cos^2\psi) \tag{21}$$

plays the main role. The factor $1-\cos\psi$ is a maximum for backward scattering; $1-\cos^2\psi$, for scattering at right angles. It is assumed, of course, that the law of force between molecules is such that these factors fall off rapidly for large values of $b$, so that the integrals in Eqs. (19) and (21) will converge.

In terms of the cross section $\sigma(V)$ defined by Eq. (19) the momentum transferred to members of the first group of molecules from members of the second group, per unit volume and unit time, can be written

$$-\mathrm{d}n_1\mathrm{d}n_2\mu\mathbf{V}V\sigma(V). \tag{22}$$

*Transfer of momentum between molecular species.*—The number densities $\mathrm{d}n_1$ and $\mathrm{d}n_2$ of our groups of molecules can conveniently be expressed in terms of velocity-distribution functions. The number density of molecules of type 1 having their three components of velocity in the ranges $u_1$ to $u_1+\mathrm{d}u_1$, $v_1$ to $v_1+\mathrm{d}v_1$, $w_1$ to $w_1+\mathrm{d}w_1$ is

$$\mathrm{d}n_1 = n_1\mathrm{f}_1(u_1, v_1, w_1)\mathrm{d}u_1\mathrm{d}v_1\mathrm{d}w_1 = n_1\mathrm{f}_1(\mathbf{c}_1)\mathrm{d}\omega_1. \tag{23}$$

Similarly,

$$\mathrm{d}n_2 = n_2\mathrm{f}_2(u_2, v_2, w_2)\mathrm{d}u_2\mathrm{d}v_2\mathrm{d}w_2 = n_2\mathrm{f}_2(\mathbf{c}_2)\mathrm{d}\omega_2. \tag{24}$$

The total densities $n_1$ and $n_2$ are, in general, functions of (macroscopic) position. The functions $\mathrm{f}_1$ and $\mathrm{f}_2$ are taken to be normalized so that

$$\int \mathrm{f}_1\mathrm{d}\omega_1 = \int \mathrm{f}_2\mathrm{d}\omega_2 = 1, \tag{25}$$

but the *forms* of these functions may, of course, depend on position. We need consider only cases in which $n_1$, $n_2$, $\mathrm{f}_1$, $\mathrm{f}_2$ are independent of $y$ and $z$, and $\mathrm{f}_1$, $\mathrm{f}_2$ depend only on the respective speeds $c_1$, $c_2$ and the $x$-components $u_1$, $u_2$:

$$n_i \equiv \mathrm{n}_i(x); \quad \mathrm{f}_i \equiv \mathrm{f}_i(c_i, u_i; x). \tag{26}$$

The concentration gradient and the net flow of each species are then parallel to the $x$-axis.

The gradient of the partial pressure $p_1$, obtained by integrating expression (22), is then directed along the $x$-axis, and by Eq. (12) we have

$$p(\partial\gamma_1/\partial x) = -\int \mathrm{d}n_1 \int \mathrm{d}n_2\mu V_x V\sigma(V). \tag{27}$$

Here $\mathrm{d}n_1$ and $\mathrm{d}n_2$ can be expressed in terms of the distribution functions $\mathrm{f}_1$, $\mathrm{f}_2$ as shown in Eqs. (23) and (24). It is convenient sometimes to use the notation of mean values, defined by

$$\langle \mathrm{F}\rangle_{\mathrm{Av}} \equiv \int \mathrm{f}_1\mathrm{d}\omega_1 \int \mathrm{f}_2\mathrm{d}\omega_2\mathrm{F}, \tag{28}$$

where F is any function of the velocity components, and $\langle \mathrm{F}\rangle_{\mathrm{Av}}$ is in general a function of position. Then Eq. (27) can be written in the form

$$p(\partial\gamma_1/\partial x) = -n_1n_2\langle \mu V_x V\sigma(V)\rangle_{\mathrm{Av}}. \tag{29}$$

### Relative Flow of Molecular Species

To determine the actual forms of the distribution functions $\mathrm{f}_1$ and $\mathrm{f}_2$ it is necessary to resort to the Enskog-Chapman theory. Elementary arguments can be conducted by adopting more or less crude forms for these functions, choosing them to fit assumed values of the relative flow of the two species and of the temperature gradient. The relative flow can be expressed in terms of the mean drift velocities[17] $\langle \mathbf{c}_1\rangle_{\mathrm{Av}}$, $\langle \mathbf{c}_2\rangle_{\mathrm{Av}}$, where, by

[17] Not to be confused with the *mean speeds* of thermal agitation, $\bar{c}_i$. The vector quantities $\langle \mathbf{c}_i\rangle_{\mathrm{Av}}$ have macroscopic significance.

Eqs. (28) and (25),

$$\langle \mathbf{c}_i \rangle_{\mathrm{Av}} = \int \mathbf{c}_i \mathbf{f}_i d\omega_i. \tag{30}$$

In the sort of case chosen for consideration, $\langle \mathbf{c}_1 \rangle_{\mathrm{av}}$ and $\langle \mathbf{c}_2 \rangle_{\mathrm{av}}$ are, of course, directed parallel to the $x$-axis. It is convenient also to suppose that the mean drift velocity of all the molecules[18] is zero:

$$n_1\langle \mathbf{c}_1 \rangle_{\mathrm{av}} + n_2\langle \mathbf{c}_2 \rangle_{\mathrm{av}} = n(\gamma_1\langle \mathbf{c}_1 \rangle_{\mathrm{av}} + \gamma_2\langle \mathbf{c}_2 \rangle_{\mathrm{av}}) = 0. \tag{31}$$

For any chosen form of $\mathbf{f}_1$ and $\mathbf{f}_2$, the calculation indicated in Eq. (27) or (29) will give the concentration gradient.

### The Diffusion Coefficients

As will be illustrated in the calculations of the following sections, and as is quite plausible in advance, the concentration gradient always turns out to be a homogeneous linear function of the quantities $\langle \mathbf{c}_1 \rangle_{\mathrm{av}}$ and $\partial T/\partial x$. The standard way of indicating this fact in writing equation of diffusion is, for cases satisfying Eqs. (26) and (31),

$$\gamma_1\langle c_{1x} \rangle_{\mathrm{av}} = -D_{12}(\partial\gamma_1/\partial x) + (D_T/T)(\partial T/\partial x). \tag{32}$$

Here $D_{12}$ is the diffusion coefficient, and $D_T$ is the coefficient of thermal diffusion. From Eqs. (29) and (32) we have

$$D_{12} = \frac{\gamma_1 p\langle u_1 \rangle_{\mathrm{av}}}{n_1 n_2 \langle \mu V_x V\sigma(V)\rangle_{\mathrm{Av}}}, \quad \frac{\partial T}{\partial x} = 0 \tag{33}$$

and

$$k_T \equiv \frac{D_T}{D_{12}} = -\frac{T n_1 n_2 \langle \mu V_x V\sigma(V)\rangle_{\mathrm{av}}}{p(\partial T/\partial x)}, \quad \langle \mathbf{c}_1 \rangle_{\mathrm{av}} = 0 \tag{34}$$

as formulas for calculating the diffusion coefficient and the "thermal diffusion ratio" $k_T$.

## 3. SIMPLE METHODS OF ESTIMATING THE ORDINARY DIFFUSION COEFFICIENT

The formulation of the Stefan-Frankel method which has just been given may not seem very elementary, since it not only involves multiple integrals but refers repeatedly to distribution functions and mean values, concepts that are not very familiar to many students. Technically, it can be called completely elementary, because it does not use the differentio-integral equation which is the foundation of the rigorous theory. The rather complicated looking formulation which we have given has its uses in showing the power of the method, when suitably applied, in coming remarkably close to the results of the rigorous theory. It should be emphasized, however, that the Stefan-Frankel method is capable of being used in the simplest conceivable way to give qualitatively good results. This will be our first example.

### Calculation of $D_{12}$ on the Crudest Assumptions

We shall proceed in a way that is analogous to the calculation of the pressure of a gas given in many first-year college textbooks. The molecules of each of the two species will be taken to be divided into groups moving parallel to the coordinate axes with fixed speeds. For the present purpose it suffices to take only one direction of motion, that parallel to the $x$-axis. Since all of the molecules of each species can act as *obstacles* to the motion of those of the other species, it is reasonable to assign approximately half of the molecules of each type to each sense of motion. The speeds will be taken to be of the order of magnitude of the speeds of thermal agitation, $\bar{c}_1$ and $\bar{c}_2$, but smaller by a factor of the order of 2 to allow for the actual obliqueness of the motions of most of the molecules; we write $s_1$, $s_2$ for such suitably chosen multiples of $\bar{c}_1$ and $\bar{c}_2$.

The general idea of a collision cross section and the calculation of the transfer of momentum per unit volume and unit time between two species of molecules can, of course, be explained well enough for the present purpose much more simply than is done in SEC. 2. Without taking the space to do this, we only give the calculation on the basis of Eq. (22).

Writing $\mathbf{j}$ for the net flow of molecules of type 1, we have, by Eq. (31),

$$\mathbf{j} = n_1\langle \mathbf{c}_1 \rangle_{\mathrm{av}} = -n_2\langle \mathbf{c}_2 \rangle_{\mathrm{av}}. \tag{35}$$

In the case considered, the vector $\mathbf{j}$ lies in the positive $x$-direction. There are various possible choices of the number densities and speeds of the

[18] This is not, in general, the same as the velocity of the center of mass of all the molecules. The mean drift velocity of all the molecules is the more useful quantity, since, for example, under our assumptions (26), with $p$=const., Eq. (31) will hold automatically if the gas mixture is contained in a stationary vessel; the velocity of the center of mass will not in general vanish under these conditions.

four sets of molecules which are consistent with Eq. (35). We deal with the two possibilities that most naturally suggest themselves.

*Flow ascribed to differences in number densities.*—If we ascribe the same speed $s_1$ to all molecules of type 1 and the same speed $s_2$ to all those of type 2, we can satisfy Eq. (35) by listing the sets as follows:

| Set No. | Type of molecule | Number density | Velocity |
|---|---|---|---|
| 1 | 1 | $\frac{1}{2}n_1+\frac{1}{2}j/s_1$ | $s_1$ |
| 2 | 1 | $\frac{1}{2}n_1-\frac{1}{2}j/s_1$ | $-s_1$ |
| 3 | 2 | $\frac{1}{2}n_2+\frac{1}{2}j/s_2$ | $-s_2$ |
| 4 | 2 | $\frac{1}{2}n_2-\frac{1}{2}j/s_2$ | $s_2$ |

Summing the four contributions of the form (22) for collisions between molecules of sets 1 and 3, 2 and 4, 1 and 4, and 2 and 3, we find that all terms cancel except those that are linear in $j$, and the result is

$$\begin{aligned}p\partial\gamma_1/\partial x = -\tfrac{1}{2}j\{&[(n_2/s_1)+(n_1/s_2)]\\ &\times\mu(s_1+s_2)^2\sigma(s_1+s_2)\\ &-[(n_2/s_1)-(n_1/s_2)]\\ &\times\mu(s_1-s_2)^2\sigma(s_1-s_2)\}.\end{aligned}\tag{36}$$

It will be supposed throughout that the molecules of the two species have equal masses or that the first species is the lighter, so that

$$m_1\leqslant m_2,\quad \bar{c}_1\geqslant\bar{c}_2,\quad s_1\geqslant s_2;\tag{37}$$

this justifies our writing $s_1-s_2$ for $|s_1-s_2|$ as an argument of $\sigma$, and makes Eqs. (36), (38) and (40) unsymmetrical in the indices $_1$ and $_2$. From Eqs. (32), (35) and (36) we now get

$$\begin{aligned}(D_{12})^{-1} &= -(\partial\gamma_1/\partial x)/(\gamma_1\langle c_{1x}\rangle_{Av})\\ &=(n^2/2p)\{[1+\gamma_1(s_1/s_2)+\gamma_2(s_2/s_1)]\\ &\times\mu(s_1+s_2)\sigma(s_1+s_2)\\ &+[1-\gamma_1(s_1/s_2)-\gamma_2(s_2/s_1)]\\ &\times\mu(s_1-s_2)\sigma(s_1-s_2)\}.\end{aligned}\tag{38}$$

To simplify this expression further, we may suppose that the masses $m_1$ and $m_2$ are not very different, so that $(s_1/s_2)$ and $(s_2/s_1)$ are approximately 1. Then

$$D_{12}\approx\frac{p}{n^2\mu\cdot 2s\sigma(2s)}=\frac{kT}{2\mu s}\cdot\frac{1}{n\sigma(2s)}.\tag{39}$$

Here the first factor is of the nature and order of magnitude of a velocity of thermal agitation, and the second is essentially a mean free path. Thus the result is of the same order of magnitude as the result of the mean-free-path approach, and has the same sort of dependence on temperature and pressure. The same is true of all the results of this section, and we shall not trouble to discuss the rest of our formulas from this point of view.

To study the dependence on concentration, with regard to which the mean-free-path approach fails so badly, we must consider Eq. (38) without supposing the masses nearly equal.[19] To obtain a simplification when $s_1/s_2$ is not restricted to values near unity, we consider the velocity dependence of $\sigma(V)$.

For *rigid spherical molecules*, $\sigma$ is a geometric quantity independent of $V$. On this assumption we obtain from Eq. (38),

$$(D_{12})^{-1}=(n^2\mu\sigma/p)s_1[1+\gamma_1+(s_2/s_1)^2\gamma_2].\tag{40}$$

Thus the dependence of $D_{12}$ on concentration is given by a factor

$$[1+\gamma_1+(m_1/m_2)\gamma_2]^{-1}.\tag{41}$$

In the extreme case, for which $m_1\ll m_2$, the coefficient is twice as large for $\gamma_1\ll 1$ as for $\gamma_2\ll 1$. This is a much weaker dependence on concentration than is given by Meyer's method, although the change is in the same direction.

For *Maxwellian molecules*, exerting forces varying as the inverse fifth power of the distance, $\sigma(V)$ is inversely proportional to $V$, as we shall show in Eq. (47). With $V\sigma(V)$ independent of $V$, Eq. (38) becomes

$$(D_{12})^{-1}=(n^2\mu/p)[V\sigma(V)].\tag{42}$$

This is exactly independent of concentration.

*Flow ascribed to differences in velocities.*—If we take the two beams of each species to have equal

[19] In any Stefan-Frankel calculation, setting $m_1=m_2$ will just suffice to make $D_{12}$ independent of concentration. It is interesting that Meyer's formula, Eq. (3), does not have this property; for equal masses but unequal molecular forces Eq. (3) in general shows a dependence on concentration, although it does give the same value for $\gamma_2\ll 1$ as for $\gamma_1\ll 1$. For example, for rigid spheres with equal masses but with the radius of the second type ten times that of the first, Meyer's $D_{12}$ has a maximum value, at $\gamma_2=1/11$, which is 1.503 times the value for $\gamma_2\ll 1$ or $\gamma_1\ll 1$. This dependence on concentration caused by dissimilarity of forces certainly contributes substantially to the "implausibility of the result" discussed at the very end of Sec. 1. It is, however, much less simple to calculate, in general, than the effect of unequal masses, and also does not provide such striking conflicts with experimental data. On the other hand, it eludes completely the usual sort of considerations of "persistence of velocities" used to patch up the Meyer formula.

number densities, we can satisfy Eq. (35) by the following scheme:

| Set No. | Type of molecule | Number density | Velocity |
|---|---|---|---|
| 1 | 1 | $\frac{1}{2}n_1$ | $s_1+j/n_1$ |
| 2 | 1 | $\frac{1}{2}n_1$ | $-s_1+j/n_1$ |
| 3 | 2 | $\frac{1}{2}n_2$ | $-s_2-j/n_2$ |
| 4 | 2 | $\frac{1}{2}n_2$ | $s_2-j/n_2$ |

The contribution of the form (22) for collisions of sets 1 and 3 is

$$-(n_1n_2/4)\mu[s_1+s_2+(j/n_1)+(j/n_2)]^2 \times\sigma[s_1+s_2+(j/n_1)+(j/n_2)]. \quad (43)$$

The contribution for sets 2 and 4 is of opposite sign and has the sign of $j$ changed. The other two contributions, from collisions between 1 and 4, and 2 and 3, are obtained[20] from the first two by changing the sign of $s_2$. Since the mean drift velocities are always minute compared to the mean speeds of thermal agitation, the contributions to $V$ depending on $j$ are minute compared to $s_1$ and $s_2$, and we can combine the first two contributions to obtain

$$-(n_1n_2/4)\mu[(\mathrm{d}/\mathrm{d}V)V^2\sigma(V)]_{V=s_1+s_2} \cdot 2[(j/n_1)+(j/n_2)]. \quad (44)$$

The sum of the other two contributions to $p(\partial\gamma_1/\partial x)$ is obtained from expression (44) by changing the sign of $s_2$. Then, multiplying the sum of all four contributions by $-n/pj$ and simplifying algebraically, we obtain the result,

$$(D_{12})^{-1}=(n^2\mu/2p)\{[(\mathrm{d}/\mathrm{d}V)V^2\sigma(V)]_{V=s_1+s_2} +[(\mathrm{d}/\mathrm{d}V)V^2\sigma(V)]_{V=s_1-s_2}\}. \quad (45)$$

This is exactly independent of concentration for any law of force and for all values of $m_2/m_1$.

The two crude calculations we have made involve two opposite, extreme assumptions about the way the net flow is to be obtained from the four beams of molecules. The second calculation gives a result that is always independent of concentration. The first gives a moderate dependence on concentration for rigid spherical molecules, and none for Maxwellian molecules. The behavior of actual gases is roughly intermediate between that to be expected for rigid spheres and that to be expected for Maxwellian molecules, and is usually much closer to the latter. Thus our very crude considerations based on the Stefan-Frankel approach indicate that $D_{12}$ is nearly independent of concentration, so that they succeed much better than Meyer's discussion based on the concept of mean free path.

[20] The signs affixed to these two contributions are determined, as in the other case, by stipulation (37).

### Exact Result for Maxwellian Molecules

For a gas composed of rigid spherical molecules the viscosity would be proportional to the square root of the absolute temperature. Maxwell's experiments[21] indicated that the coefficient of viscosity of air is very nearly proportional to the absolute temperature. Maxwell decided that, since gases ought to behave simply, the proportionality should be exact. This would mean that the mean free path itself should be proportional to the square root of the absolute temperature; in other words, that collision cross sections should be inversely proportional to relative velocity. From this Maxwell concluded that the law of force must be that of the inverse fifth power, and based on this his famous treatment of kinetic theory.[22]

Maxwell's discussion of the behavior of collision cross sections for inverse-power forces was based on the actual solution of the dynamical problem, but Frankel[6] pointed out that the results could be obtained easily by a dimensional argument. The problem of solving the equations of motion for the force law

$$F=K/r^\nu \quad (46)$$

and then evaluating an integral such as those in Eqs. (19) and (21) is a well-defined one, and must give $\sigma(V)$, with dimensions $[\mathrm{L}^2]$, in terms of just three quantities: the force constant $K$, with dimensions $[\mathrm{ML}^{\nu+1}\mathrm{T}^{-2}]$; the reduced mass $\mu$, with dimensions $[\mathrm{M}]$; and the relative speed $V$, with dimensions $[\mathrm{LT}^{-1}]$. It follows at once that

$$\sigma(V)=\mathrm{const.}\cdot(K/\mu V^2)^{2/(\nu-1)}. \quad (47)$$

Thus

$$V\sigma(V)\propto V^{(\nu-5)/(\nu-1)}. \quad (48)$$

For "Maxwellian" molecules, with $\nu=5$, $V\sigma(V)$

[21] Reference 9, p. 1.

[22] Reference 9, p. 26. Five years later—in 1871 (reference 9, p. 343)—Maxwell presented a number of results based on the hypothesis of rigid spherical molecules, having evidently realized that actual molecules are not strictly "Maxwellian" and that the behavior of most actual gases is roughly intermediate between these two cases.

is independent of $V$. For "harder" molecules, with $\nu > 5$, $V\sigma(V)$ increases as $V$ increases. Rigid spherical molecules can be included formally as the case $\nu \rightarrow \infty$, with $\sigma(V)$ independent of $V$.

For the Maxwellian case the factor $V\sigma(V)$ can be removed from the quantity to be averaged in Eq. (29), and we find without approximation, taking account of Eq. (13),

$$p(\partial\gamma_1/\partial x) = -n_1 n_2 \mu[V\sigma(V)] \times[\langle c_{1x}\rangle_{\text{Av}} - \langle c_{2x}\rangle_{\text{Av}}]. \quad (49)$$

By Eq. (35) this reduces to

$$p(\partial\gamma_1/\partial x) = -nj\mu[V\sigma(V)]. \quad (50)$$

Multiplying, as usual, by $-n/pj$, we get

$$(D_{12})^{-1} = (n^2\mu/p)[V\sigma(V)]. \quad (51)$$

This result, which is independent of concentration, was obtained without making any assumption about the distribution functions $f_1$ and $f_2$, and hence holds exactly. The possibility of treating transport phenomena in the case $\nu = 5$ without the necessity of determining distribution functions gave Maxwell's treatment of kinetic theory its great elegance.

Since Eq. (50) was derived without any stipulation other than Eq. (35) about the distribution functions, it is true exactly even for nonuniform temperature of the gas mixture. Equation (50) is just Eq. (32) without the last term. Thus for Maxwellian molecules the phenomenon of thermal diffusion does not appear.

### Use of Approximately Correct Distribution Functions

We now return to the general case, in which the evaluation of the right-hand member of Eq. (29) requires the use of definite distribution functions. Instead of the very crude picture of the distribution functions used in the first calculations of this section, we shall now use the best simple assumption available.

For $\mathbf{j} = 0$, the distribution functions are known exactly, being just Maxwell's distribution law,

$$f_i^{(0)}(c_i) = (\beta_i/\pi)^{\frac{3}{2}} e^{-\beta_i c_i^2}, \quad i = 1, 2 \quad (52)$$

with

$$\beta_i = m_i/2kT. \quad (53)$$

The distributions for $\mathbf{j} \neq 0$ differ from these only very slightly, since the mean drift velocities are extremely small compared to the mean speeds of thermal agitation. We shall, accordingly, make simple small changes in Eqs. (52) so as to secure the net flow $\mathbf{j}$ defined by Eq. (35), again taking $\mathbf{j}$ to lie in the positive $x$-direction. The resulting distributions will not be the true ones, but will differ from them only slightly. Since even our very crude assumptions gave fairly good results, the present procedure seems well worth trying.

We set

$$f_1 = (1 + a_1 u_1) f_1^{(0)}. \quad (54)$$

The correction term is an odd function of $u_1$, and hence does not affect the normalization. By Eq. (35) we have

$$\mathbf{j} = n_1 \int f_1 u_1 d\omega_1 = a_1 n_1 \int f_1^{(0)} u_1^2 d\omega_1$$

$$= a_1 n_1 \langle u_1^2 \rangle_{\text{av}} = \tfrac{1}{3} a_1 n_1 \langle c_1^2 \rangle_{\text{av}} = a_1 n_1 kT/m_1. \quad (55)$$

Substituting the resulting value of $a_1$ in Eq. (54), we have

$$f_1 = [1 + (m_1 j/n_1 kT) u_1] f_1^{(0)}. \quad (56)$$

Similarly,

$$f_2 = [1 - (m_2 j/n_2 kT) u_2] f_2^{(0)}. \quad (57)$$

Since the total net flow of molecules is zero, it follows from equipartition of energy that there is no net flow of energy, that is, no thermal conduction. Thus the assumption that Eqs. (56) and (57) hold for the element of volume under consideration is consistent with the assumption that there is no temperature gradient.

The numerator of the expression for $D_{12}$ in Eq. (33) is

$$\gamma_1 pj/n_1 = pj/n = jkT, \quad (58)$$

and the denominator is

$$n_1 n_2 \mu \int f_1 d\omega_1 \int f_2 d\omega_2 \cdot (u_1 - u_2) V\sigma(V). \quad (59)$$

Since the value of $V$ is left unchanged when the signs of both $u_1$ and $u_2$ are changed, the only parts of the integrand that give nonvanishing integrals are those linear in $j$ and quadratic in $u_1$ and $u_2$. Expression (59) can accordingly be

written

$$(\mu j/kT)\int f_1^{(0)}d\omega_1\int f_2^{(0)}d\omega_2(u_1-u_2)$$
$$\times(n_2m_1u_1-n_1m_2u_2)V\sigma(V). \quad (60)$$

This expression can be simplified by introducing as variables of integration the velocity $\mathbf{C}$ of the center of mass and the relative velocity $\mathbf{V}$,

$$\mathbf{c}_1=\mathbf{C}+(m_2/M)\mathbf{V},\quad \mathbf{c}_2=\mathbf{C}-(m_1/M)\mathbf{V}, \quad (61)$$

where $M$ is an abbreviation for $m_1+m_2$. Then we have[23]

$$m_1c_1^2+m_2c_2^2=MC^2+\mu V^2. \quad (62)$$

Also

$$n_2m_1u_1-n_1m_2u_2=(n_2m_1-n_1m_2)C_x+n\mu V_x. \quad (63)$$

We have

$$du_1du_2=\begin{vmatrix}1 & -m_1/M\\ 1 & m_2/M\end{vmatrix}dC_xdV_x=dC_xdV_x, \quad (64)$$

so that

$$d\omega_1d\omega_2=dC_xdC_ydC_zdV_xdV_ydV_z. \quad (65)$$

If we make the definitions

$$B\equiv M/2kT,\quad \beta\equiv\mu/2kT, \quad (66)$$

we see that

$$\beta_1\beta_2=B\beta, \quad (67)$$

and also, by Eq. (62), that

$$\beta_1c_1^2+\beta_2c_2^2=BC^2+\beta V^2. \quad (68)$$

Expression (60) then becomes

$$(\mu j/kT)(\beta/\pi)^{\frac{3}{2}}$$
$$\times\iiint e^{-\beta V^2}dV_xdV_ydV_z(B/\pi)^{\frac{3}{2}}$$
$$\times\iiint e^{-BC^2}dC_xdC_ydC_z$$
$$\cdot V_x[(n_2m_1-n_1m_2)C_x+n\mu V_x]V\sigma(V). \quad (69)$$

Here the term in $C_x$ is an odd function of $C_x$ and contributes nothing to the integral. The other term depends on $\mathbf{C}$ only through the factor $(B/\pi)^{\frac{3}{2}}e^{-BC^2}$, which is a normalized function like those defined in Eq. (52), so that its integral over the $C$-space gives the factor unity. By symmetry we can replace $V_x^2$ by $\frac{1}{3}V^2$, so that our expression reduces to

$$(n\mu^2j/3kT)(\beta/\pi)^{\frac{3}{2}}$$
$$\times\iiint e^{-\beta V^2}dV_xdV_ydV_z\cdot V^3\sigma(V). \quad (70)$$

We now integrate over the direction of the vector $\mathbf{V}$:

$$dV_xdV_ydV_z=V^2dV\sin\theta_Vd\theta_Vd\phi_V; \quad (71)$$

$$\iiint f(V)dV_xdV_ydV_z=4\pi\int_0^\infty V^2f(V)dV. \quad (72)$$

Writing also

$$\beta^{\frac{1}{2}}V\equiv g, \quad (73)$$

we find that the formula of Eq. (33), which has (58) for its numerator and (70) for its denominator, has become

$$D_{12}=3kT/16\mu n\Omega_{12}^{(1)}(1), \quad (74)$$

where

$$\Omega_{12}^{(1)}(r)=2^{-1}\pi^{-\frac{1}{2}}\int_0^\infty e^{-g^2}g^{2r+2}V\sigma(V)dg, \quad (75)$$

with $V$ related to $g$ by Eqs. (73) and (66). This is the notation used by Chapman and Cowling.[24]

Equation (74) makes $D_{12}$ exactly independent of concentration. What is more, Eq. (74) is just the formula given by the first approximation of the rigorous Chapman-Enskog theory. The evaluation of the second approximation in some cases, and of higher approximations in very special cases, shows that the first approximation in general gives very nearly the correct result.

For Maxwellian molecules the particular forms postulated for $f_1$ and $f_2$ do not matter, and Eqs. (42), (45) and (74) all agree with the exact result of Eq. (51). For rigid spherical molecules our crude results given in Eqs. (40) and (45) agree fairly well with the good result of Eq. (74). As already mentioned, the first of these crude results shows a false dependence on concentration,

[23] This is the theorem that the total kinetic energy is equal to the kinetic energy of the total mass, moving with the speed of the center of mass, plus the kinetic energy of relative motion.

[24] Reference 2, p. 157.

though not a very strong one. Besides this, both of the crude results for rigid spherical molecules show an asymmetric dependence on the molecular weights, because the oversimplified velocity distributions allow only one kind of molecule to make "overtaking" collisions. The asymmetry can be expressed by a factor $[1+(m_1/m_2)]^{\frac{1}{2}}$, which never differs greatly from unity.

For the case of rigid spherical molecules Stefan obtained a result equivalent to Eq. (74) by using essentially the same argument as has been given here. The result given by Maxwell for rigid spherical molecules differs from Stefan's by the factor $2\sqrt{2}/3$, or by about 6 percent. It is not easy to guess how this difference in numerical factor came about.

The great superiority of Stefan's approach over Meyer's as regards physical principles is shown most strikingly by what happens when one postulates that Maxwell's inverse fifth-power law of force holds between the molecules. When this is done in Meyer's method, the only effect is to make $D_{12}$ proportional to $T^2$, rather than to $T^{\frac{3}{2}}$, as it is for rigid spherical molecules; the method remains a crude one, and difficulties with the dependence on concentration remain. On the other hand, when we introduce the inverse fifth-power law into Stefan's method we note at once that the necessity for guessing at the velocity distributions, which is the one difficulty with the method, is removed, and we are led to claim that the result obtained holds exactly. This claim is borne out by the rigorous Enskog-Chapman theory. The calculations by Stefan's method for this case are essentially just a simple restatement, in more physical language, of the part of Maxwell's famous 1866 theory which deals with diffusion.

## 4. THE ELEMENTARY EXPLANATION OF THERMAL DIFFUSION

### General Qualitative Considerations

A main achievement of Frankel's argument about thermal diffusion is the demonstration that the effect vanishes for Maxwell's inverse fifth-power law. This has already been brought out in the preceding section. When $V\sigma(V)$ is independent of $V$ we have, by Eq. (29),

$$\begin{aligned} p(\partial\gamma_1/\partial x) &= -n_1n_2\langle\mu V_x V\sigma(V)\rangle_{\text{av}} \\ &= -n_1n_2\mu[V\sigma(V)]\langle V_x\rangle_{\text{av}} \\ &= -n_1n_2\mu[V\sigma(V)][\langle u_1\rangle_{\text{av}}-\langle u_2\rangle_{\text{av}}]. \end{aligned} \tag{76}$$

The gradient of concentration accordingly depends only on the relative flow of the two species, and is unaffected by a temperature gradient.

Frankel also remarks that if the intermolecular force depends on distance more or less strongly than by the inverse fifth power, a temperature gradient will produce an effect which will be of one sign if the molecules are "harder" than Maxwellian molecules and of the opposite sign if they are "softer." The discussion can be confined to the usual case of "hard" molecules, in which the force varies with distance more strongly than by the inverse fifth power. Then $V\sigma(V)$ increases as $V$ increases, and there is a disproportionately large transfer of momentum in the collisions in which $V$ is larger.

For the case in which the molecules differ in mass, Frankel argues that, while both kinds of molecule have higher average speeds when coming from the hotter part of the gas than when coming from the colder part, the difference is greater for the lighter molecules than for the heavier ones. Thus more of the collisions with large values of $V$ will have the lighter molecule coming from the hotter region rather than vice versa, and the net transfer of momentum from heavy to light molecules will be directed toward the hotter part of the gas. This means that the gradient of partial pressure, $p(\partial\gamma_1/\partial x)$, will include a part proportional to $(\partial T/\partial x)$, with a positive coefficient, besides the term proportional to the relative flow of the two species. Thus—speaking always of the case of "hard" molecules—there will be an effect of thermal diffusion acting to produce an increased concentration of the lighter species of molecules in the hotter region. Since species 1 is taken to be the lighter species, we see that $D_T$ as defined in Eq. (32) is positive.

It is easy to extend the argument to cover cases in which the two kinds of molecule differ otherwise than in their masses. Suppose that one species has a longer mean free path in the gas mixture than the other species. Then the difference in average speeds of molecules coming from

the hotter and colder regions will be greater for the species having the longer mean free path. Thus, by the same argument that Frankel gives for the effect of a difference of mass, there will be an effect of thermal diffusion acting to increase the concentration in the hotter region of the molecular species which has the longer mean free path.

The obvious reason for a difference of mean free paths is that one species of molecules may be "smaller" than the other, in the sense that the effective range of the force it exerts is less. Then thermal diffusion will act to concentrate the smaller molecules in the hotter region. The coefficient $D_T$ in Eq. (32) will be positive if species 1 has the smaller molecules.

Another possible source of a difference of mean free paths is that the effective collision cross section for unlike molecules may be different from that for like molecules of either species. If the cross section for unlike molecules is smaller than that for like molecules, the species with the smaller concentration will have the longer mean free path, and thermal diffusion will act to increase the concentration of this species in the hotter region. Since, as shown in Eq. (32), the thermal diffusion coefficient is defined so that it is positive when the effect is to increase the concentration of species 1 in the hotter region,[25] it will in this case be proportional to $\gamma_2-\gamma_1$. If the cross section is greater for unlike molecules than for like, the coefficient will be proportional to $\gamma_1-\gamma_2$.

The first suggestion of the possibility of an effect of this sort seems to have been made by H. M. Mott-Smith,[26] who considered, for the case of isotopic mixtures, the quantum-mechanical effect of the symmetry properties of the wave functions for identical particles. This effect would usually influence thermal diffusion only slightly, and no observations confirming it have been published. In the case of strongly dissimilar molecules the cross section for like molecules of one species may be very different from that for like molecules of the other species, and elementary considerations are not capable of yielding a very good estimate of the part of the thermal diffusion coefficient that depends on the difference of concentration. A discussion based on the exact theory has been given by Chapman.[27]

An interesting case which has been investigated experimentally is that of neon-ammonia mixtures.[28] If we call the lighter molecules, those of ammonia, species 1, the effect of the difference of masses makes a positive contribution to $D_T$, but, since the molecules of ammonia are considerably larger than those of neon, the effect of the difference of sizes makes a negative contribution. These two effects largely cancel, but the latter effect is the larger, so that usually $D_T$ is negative, and the neon is concentrated in the hotter region. When the mixture contains more than 75 percent neon, however, the sign of $D_T$ changes and it becomes positive. The partial cancellation of the other two effects makes possible the observation of a contribution proportional to $\gamma_2-\gamma_1$.

Frankel remarked that it is clear, if his approach is used, that the thermal diffusion coefficient is proportional to the product of the concentrations. This is obvious, for example, from a glance at Eq. (34). General qualitative considerations about the velocity distributions have now yielded the following conclusions about the thermal diffusion coefficient:

(1) It vanishes for Maxwellian molecules and, *ceteris paribus*, will have one sign for "harder" molecules, the other for "softer" molecules.

(2) Its main dependence on the concentrations is through a factor $\gamma_1\gamma_2$, which does not affect the sign.

(3) For "hard" molecules, the larger mass of the molecules of species 2 gives rise to a positive contribution to $D_T$.

(4) A difference in size of the molecules gives rise to a contribution to $D_T$. For "hard" molecules this contribution is positive if the molecules of species 2 are the larger ones.

(5) A difference in collision cross sections for like and unlike molecules gives rise to a contribution to $D_T$ proportional to the difference of the concentrations. For "hard" molecules this contribution is proportional to $+(\gamma_2-\gamma_1)$ if the cross section for unlike molecules is the greater.

The first two of these conclusions[29] are the most important, and are valid in general. The

[25] It should be mentioned that the opposite convention is also often used.

[26] Private communication, April 1940.

[27] S. Chapman, *Proc. Roy. Soc.* **177A**, 38 (1940).

[28] K. E. Grew, *Nature* **150**, 320 (1942).

[29] Rai and Kothari (reference 5) obtain conclusions (2), (3), (4) and (5) by algebraic manipulation of a simplified mean-free-path formalism which is in at least one respect incorrect (see footnote 13). They give no physical reasoning to explain these results. Manipulations can accomplish a great deal, but conclusion (1) remains completely inaccessible by the mean-free-path approach.

special contributions described in the other three conclusions are clearly distinguishable only if the properties of the two species of molecules are not very different. For very dissimilar molecules the dependence on concentration becomes considerably more complicated, and cannot be discussed adequately by elementary methods. The situation in this regard is like that met with in connection with the viscosity and thermal conductivity of mixtures of highly dissimilar gases.

The information which we have obtained by discussing the physics of thermal diffusion comprises all the valid information that we could hope to obtain by any calculation conducted on an elementary basis. This fact must be regarded as a distinct virtue of Frankel's approach. An approximate formula for the thermal diffusion ratio for binary mixtures of not too dissimilar gases can be written down without any further observation except that the ratio $k_T$ is, by definition—Eq. (34)—a dimensionless quantity. We have then

$$\begin{aligned} k_T \equiv D_T/D_{12} \approx \gamma_1\gamma_2[(\nu-5)/(\nu-1)] \\ \times\{C_1(m_2-m_1)/(m_2+m_1) \\ +C_2(\sigma_{22}-\sigma_{11})/(\sigma_{22}+\sigma_{11}) \\ +C_3(\gamma_2-\gamma_1)[1-2\sigma_{12}/(\sigma_{22}+\sigma_{11})]\}. \end{aligned} \tag{77}$$

Here $\sigma_{22}$, $\sigma_{11}$, $\sigma_{12}$ are collision cross sections, the subscripts indicating the types of molecule involved, and $C_1$, $C_2$, $C_3$ are numbers of the order of magnitude of unity. The factor involving $\nu$ and giving expression to the first of our five conclusions is obtained by referring to Eq. (48). The actual dependence on $\nu$ is, of course, more complicated, but the factor written gives the only sensitive dependence.

The so-called thermal diffusion constant,

$$\alpha \equiv k_T/\gamma_1\gamma_2, \tag{78}$$

is often used in calculations on separation of isotopes by thermal diffusion. It is not independent of concentration unless the molecules are somewhat similar and the term in $\gamma_2-\gamma_1$ in Eq. (77) is negligible. It may usually be expected to depend to some extent on the temperature, but is independent of pressure.

## Calculation from Assumed Velocity Distributions

Although all of the reliable information about thermal diffusion that can be obtained by an elementary approach has already been found above without any actual calculations, there is a certain interest in carrying such a calculation through and seeing how expression (34) for $k_T$ actually reduces to the form of Eq. (77).

In this sort of calculation the idea of mean free path must be used, as in the discussion already given. The Stefan-Frankel approach eliminates the idea of mean free path from the treatment of ordinary diffusion, but it still has to be used in dealing with thermal diffusion, just as for thermal conductivity. The method actually puts the elementary treatment of ordinary diffusion on a much better basis than any of the other transport phenomena, but leaves the elementary treatment of thermal diffusion subject to just the same difficulties and inaccuracies as those of thermal conductivity and viscosity.

The calculations can be based either on a one-dimensional picture of four "beams" of molecules, like our crudest treatments of ordinary diffusion, or on the use of suitably modified Maxwellian distributions of velocities. Either procedure is capable of giving all the information obtainable by elementary means, namely, Eq. (77). Both sorts of calculation have been carried through with care by Cacciapuoti[7] for the case of rigid spherical molecules that differ in mass only.

The crude one-dimensional treatment will be omitted here in favor of a three-dimensional treatment, which is somewhat simpler than Cacciapuoti's. This treatment will show that Frankel's approach is capable of giving precisely the factor by which the main dependence on the law of molecular force is determined in the rigorous theory; this contradicts a previous statement by the writer.[30] It will also take into account the simultaneous occurrence of both ordinary diffusion and thermal diffusion, and thus lead to the complete equation of diffusion, Eq. (32).

The approximate velocity-distribution functions are taken in the form

$$\mathrm{f}_i=\mathrm{f}_i^{(0)}(1+a_iu_i-b_iu_ic_i^2), \quad i=1,2, \tag{79}$$

where the $\mathrm{f}_i^{(0)}$ are the Maxwellian distributions defined by Eq. (52). The net flow of molecules of species 1 in the positive $x$-direction is

$$\begin{aligned} j=n_1\int u_1\mathrm{f}_1d\omega_1 &= n_1(\tfrac{1}{3}a_1\langle c_1^2\rangle_{\mathrm{av}}-\tfrac{1}{3}b_1\langle c_1^4\rangle_{\mathrm{av}}) \\ &= n_1[a_1(kT/m_1)-5b_1(kT/m_1)^2]. \end{aligned} \tag{80}$$

Similarly, by Eq. (35), we set

$$j=-n_2\int u_2\mathrm{f}_2d\omega_2=n_2[-a_2(kT/m_2)+5b_2(kT/m_2)^2]. \tag{81}$$

The transfers of kinetic energy in the negative $x$-direction

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[30] R. C. Jones and W. H. Furry, *Rev. Mod. Physics* 18, 156 (1946). This paper was delayed in publication, and the statement in question is one in which the writer concurred in 1940. The more general statement that "the weakness of Frankel's argument is that it cannot take into consideration the details of the asymmetry of the velocity distribution function itself" remains true; this is, however, also just the weakness of the elementary treatments of thermal conductivity and viscosity.

by molecules of types 1 and 2 are

$$-q_{1x}=-n_1\int u_1\cdot\tfrac{1}{2}m_1c_1^2\mathrm{f}_1\mathrm{d}\omega_1=-n_1m_1(\tfrac{1}{6}a_1\langle c_1^4\rangle_{\mathrm{av}}-\tfrac{1}{6}b_1\langle c_1^6\rangle_{\mathrm{av}})$$
$$=-(n_1m_1/2)[5a_1(kT/m_1)^2-35b_1(kT/m_1)^3], \tag{82}$$
$$-q_{2x}=-(n_2m_2/2)[5a_2(kT/m_2)^2-35b_2(kT/m_2)^3]. \tag{83}$$

From Eqs. (80) to (83) we get

$$-q_{1x}=-j\cdot\tfrac{5}{2}kT+5b_1n_1m_1(kT/m_1)^3, \tag{84}$$
$$-q_{2x}=j\cdot\tfrac{5}{2}kT+5b_2n_2m_2(kT/m_2)^3. \tag{85}$$

Here the terms proportional to $j$ represent the kinetic energy transferred by the net flows of molecules of the two species;[31] since the total flow of molecules is taken to be zero these two terms are equal but of opposite signs. The last terms of Eqs. (84) and (85) must be attributed to the thermal conductivity of the gas mixture.

The coefficient of thermal diffusion will turn out to be proportional to the difference of two terms, one containing the factor $b_1$ and the other $b_2$. Since neither the rigorous theory[32] nor experimental results provide any way of separating the thermal conduction of a gas mixture into parts to be attributed to its constituents, we are obliged to choose values for $b_1$ and $b_2$ by means of crude considerations based on the idea of mean free path. The choice of these expressions, particularly as regards numerical factors, is partly a matter of taste. We set

$$5b_in_im_i(kT/m_i)^3=\tfrac{5}{2}\cdot\frac{3k}{2m_i}\cdot\tfrac{1}{3}n_im_i\bar{c}_i\lambda_i(\partial T/\partial x). \tag{86}$$

The choice of factors is based on the approximate relations that hold for each gas taken separately:

thermal conductivity $\cong\tfrac{5}{2}c_v\cdot$(coefficient of viscosity),
$c_v=3k/2m_i$, (monatomic gas)
coefficient of viscosity $\approx\tfrac{1}{3}n_im_i\bar{c}_i\lambda_i$,

where $\lambda_i$ is the mean free path. The numerical factor $\tfrac{1}{3}$ is perhaps as well justified as any.

From Eqs. (80) to (86) we obtain

$$a_1=(m_1j/n_1kT)+(5kT/m_1)b_1, \tag{87}$$
$$a_2=-(m_2j/n_2kT)+(5kT/m_2)b_2, \tag{88}$$
$$b_1=(m_1^2\bar{c}_1\lambda_1/4k^2T^3)(\partial T/\partial x), \tag{89}$$
$$b_2=(m_2^2\bar{c}_2\lambda_2/4k^2T^3)(\partial T/\partial x). \tag{90}$$

From Eqs. (28) and (29) we have

$$nkT(\partial\gamma_1/\partial x)=-n_1n_2\mu\int\mathrm{f}_1\mathrm{d}\omega_1\int\mathrm{f}_2\mathrm{d}\omega_2(u_1-u_2)V\sigma(V). \tag{91}$$

On substituting Eq. (79) and dropping from the integrand terms that change sign when the directions of $\mathbf{c}_1$ and $\mathbf{c}_2$ are reversed, we have

$$nkT(\partial\gamma_1/\partial x)=-n_1n_2\mu\int\mathrm{f}_1^{(0)}\mathrm{d}\omega_1\int\mathrm{f}_2^{(0)}\mathrm{d}\omega_2$$
$$\times(a_1u_1+a_2u_2-b_1u_1c_1^2-b_2u_2c_2^2)(u_1-u_2)V\sigma(V). \tag{92}$$

The calculation of the terms in $a_1$ and $a_2$ is the same as that made in the last part of Sec. **3**, and the terms in $b_1$ and $b_2$ can readily be evaluated in the same way. On making the transformation described in Eqs. (61) to (68) and performing the integration over $\mathrm{d}C_x\mathrm{d}C_y\mathrm{d}C_z$, we get, from Eq. (92),

$$nkT(\partial\gamma_1/\partial x)=-n_1n_2\mu(\beta/\pi)^{3/2}$$
$$\times\int e^{-\beta V^2}\cdot4\pi V^2\mathrm{d}V\cdot V\sigma(V)$$
$$\times\{[V^2(a_1m_2-a_2m_1)/3M]$$
$$-[5V^2kT(b_1m_2-b_2m_1)/3M^2]$$
$$-[V^4(b_1m_2^3-b_2m_1^3)/3M^3]\}. \tag{93}$$

The right-hand member of Eq. (93) can at once be written in terms of the quantities $\Omega_{12}^{(1)}(r)$ of the Chapman-Cowling notation, defined by Eq. (75). We then have

$$nkT(\partial\gamma_1/\partial x)=-n_1n_2(16kT/3)(m_2+m_1)^{-2}$$
$$\times\{[(a_1m_2-a_2m_1)(m_1+m_2)$$
$$-5kT(b_1m_2-b_2m_1)]\Omega_{12}^{(1)}(1)$$
$$-(2kT/m_1m_2)(b_1m_2^3-b_2m_1^3)\Omega_{12}^{(1)}(2)\}. \tag{94}$$

When the values of $a_1$, $a_2$, $b_1$, $b_2$ given in Eqs. (87) to (90) are substituted in Eq. (94), this equation becomes

$$nkT(\partial\gamma_1/\partial x)=-(16n\mu j/3)\Omega_{12}^{(1)}(1)-(4n_1n_2\mu^2/3)$$
$$\times[(\bar{c}_1\lambda_1/m_1)-(\bar{c}_2\lambda_2/m_2)]$$
$$\times[5\Omega_{12}^{(1)}(1)-2\Omega_{12}^{(1)}(2)]T^{-1}(\partial T/\partial x). \tag{95}$$

This is the same as the general equation of diffusion, Eq. (32), in virtue of Eq. (35) and the identifications

$$D_{12}=\frac{3kT}{16\mu n\Omega_{12}^{(1)}(1)}, \tag{96}$$
$$k_T\equiv(D_T/D_{12})=(4n\mu^2/3kT)\gamma_1\gamma_2$$
$$\times[2\Omega_{12}^{(1)}(2)-5\Omega_{12}^{(1)}(1)][(\bar{c}_1\lambda_1/m_1)-(\bar{c}_2\lambda_2/m_2)]. \tag{97}$$

The first of these is, of course, the same as Eq. (74).

The resulting expression for $k_T$ is proportional to $\gamma_1\gamma_2$. In the first brackets it contains the same factor as occurs in the result of the rigorous theory[33] to determine the strongest dependence of $k_T$ on the force law. From Eqs. (48), (73) and (75) we can evaluate this factor:

$$2\Omega_{12}^{(1)}(2)-5\Omega_{12}^{(1)}(1)=2^{-1}\pi^{-\frac{1}{2}}[V\sigma(V)]_{g=1}$$
$$\times\int_0^\infty e^{-g^2}(2g^6-5g^4)g^{(\nu-5)/(\nu-1)}\mathrm{d}g$$
$$=\{\Gamma[3-2/(\nu-1)]/4\pi^{\frac{1}{2}}\}$$
$$\cdot[(\nu-5)/(\nu-1)][V\sigma(V)]_{g=1}$$
$$\cong(kT/8\mu)^{\frac{1}{2}}\sigma[(2kT/\mu)^{\frac{1}{2}}][(\nu-5)/(\nu-1)]. \tag{98}$$

In the last form written the insensitive factor involving the gamma-function, which increases by only about 50 percent as $\nu$ changes from 5 to $\infty$, has been replaced by a constant.

The main dependences of $k_T$ on the force law and on concentration are, accordingly, given correctly by the first

[31] The occurrence of the factor 5/2, rather than the 3/2 which might at first thought be expected, is due to the fact that a larger fraction of the faster molecules than of the slower ones passes through unit area in unit time (Enskog, reference 1, p. 80). The discussion with regard to the factor 5/2 given by Chapman and Cowling (reference 2, p. 145, footnote) seems to put the emphasis on the wrong part of Enskog's remarks.

[32] The first approximation to the thermal conductivity is the ratio of two quadratic functions of the concentrations (Chapman and Cowling, reference 2, p. 166).

[33] Chapman and Cowling, reference 2, p. 167; the meaning of abbreviations is explained on p. 164.

factors of Eq. (97). For the dependence on the differences of molecular properties and for any further dependences on concentration we must examine the last factor. This factor, which involves the mean free paths, cannot be regarded as giving reliable information when the molecules are very dissimilar. We shall deal with it only for the case in which the differences between the molecules are small. In this case, if we write for $\bar{c}_1$ and $\bar{c}_2$ the root-mean-square values, in order to have convenient numerical factors, we find

$$(\bar{c}_1\lambda_1/m_1)-(\bar{c}_2\lambda_2/m_2)=(3kT)^{\frac{1}{2}}[(\lambda_1/m_1^{\frac{1}{2}})-(\lambda_2/m_2^{\frac{1}{2}})]$$
$$\cong(3kT/m^3)^{\frac{1}{2}}\lambda[3(m_2-m_1)/(m_2+m_1)+(\lambda_1-\lambda_2)/\lambda]. \quad (99)$$

Here $m$ and $\lambda$ are mean values, from which the values for the two species of molecules differ only slightly. Then $m\cong 2\mu$.

Substitution of Eqs. (98) and (99) in Eq. (97) gives

$$k_T\equiv(D_T/D_{12})=(\lambda n\sigma/2\sqrt{3})\gamma_1\gamma_2[(\nu-5)/(\nu-1)]$$
$$\cdot[3(m_2-m_1)/(m_2+m_1)+(\lambda_1-\lambda_2)/\lambda]. \quad (100)$$

Here $\sigma(V)$ is to be evaluated at the argument shown in Eq. (98), which is a typical value of $V$. Then $\lambda n\sigma$ is of the order of magnitude unity, and numerical factors are, of course, not to be taken seriously. When the term $(\lambda_2-\lambda_1)/\lambda$ is expressed approximately in terms of differences of molecular properties, the result takes on precisely the form of Eq. (77).

## 5. CONCLUSION

The traditional elementary theory based on the idea of mean free path succeeds very well in the treatment of viscosity and thermal conductivity of single gases; the results are really uncertain only as to numerical constant factors. For mixtures of strongly dissimilar gases the dependences of these effects on concentration are not given accurately by the elementary treatment, but there are no very large discrepancies.

In the case of diffusion the mean-free-path approach gives results as good as those for viscosity and thermal conductivity in only one case —that in which the differences between the two molecular species are practically negligible. This is the classical problem of "self-diffusion," and is well realized in most cases in which the differences are isotopic only, not chemical. As we pointed out at some length, the mean-free-path theory encounters serious difficulties in the problem of diffusion of strongly dissimilar gases. The more difficult problem of thermal diffusion presents questions with which the mean-free-path theory is completely unable to deal.

The approach used by Stefan for the problem of ordinary diffusion makes it possible to find by the crudest sort of arguments results that are quite as good as the usual elementary results for viscosity and thermal conductivity. When rather more care is used, Stefan's method gives precisely the result of the first approximation of the rigorous theory, which is very accurate in nearly all cases.

Frankel's treatment of thermal diffusion combines Stefan's procedure with some arguments of the mean-free-path variety. It seems fair to ascribe the remaining inaccuracies of Frankel's theory to the latter ingredient, and credit its general success to the distinctly superior qualities of Stefan's approach. The combination devised by Frankel makes it possible to obtain results which, for mixtures of slightly dissimilar components, are of just the same quality as the traditional elementary theory of viscosity and thermal conductivity of single gases; namely, the only important uncertainties are in the values of numerical constants [see Eq. (77)]. For mixtures of highly dissimilar gases it becomes impossible to check the dependence on concentration in detail, just as in the elementary theory of viscosity and thermal conductivity.

*The sages do not consider that making no mistakes is a blessing. They believe, rather, that the great virture of man lies in his ability to correct his mistakes and continually to make a new man of himself.*—WANG YANG-MING.

## Servomechanisms

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In research problems and in industrial processes, there is frequently needed a device for holding some physical quantity at a constant value despite fluctuations in the operating conditions that tend to alter this desired value; in other problems, it may be desired that the physical quantity vary in a fixed, predetermined manner, independently of fluctuations in operating conditions. Some of the physical quantities whose control is frequently desired are position, velocity, acceleration, voltage, current, magnetic field intensity, temperature, pressure, tension, and so forth. Thus, the navigator aboard a ship at sea might desire a device for holding his sextant steady against the roll, pitch and yaw of the ship; the antiaircraft battery on this ship needs a device that will cause the guns to follow the target without regard to the motions of the ship itself. The operator of a cyclotron or of a mass spectrograph would like a device for keeping the magnetic field intensity constant despite fluctuations in the power supply and the effects of temperature changes in the exciting coils and the pole pieces.

Devices for producing such results are called by the generic term *controller*. For our purposes, controllers may be divided into two kinds: open-cycle and closed-cycle controllers. A servomechanism is a type of closed-cycle controller. The difference between an open-cycle and a closed-cycle controller may be illustrated in the following examples.

Consider an automobile whose speed is controlled by the position of a button or pedal. For each fixed position of the pedal there will be a definite speed of the car, but this speed will also be a function of the quality of the fuel, the condition of the tires and of the road, the angle of ascent or descent on a hill, the temperature of the cooling system, and so forth. Such a pedal and throttle is an example of an open-cycle controller.

Suppose, now, we insert between the pedal and the throttle a mechanism that automatically adjusts the throttle whenever the speed either exceeds or falls below the speed corresponding to the given position of the pedal under some standard set of conditions. We would then have a closed-cycle controller.

Or, consider two possible ways of driving an astronomical telescope: first, by a synchronous motor actuated by a high precision frequency standard; second, by photoelectric devices actuated by the star being followed, and controlling the driving motor. The first method is an example of open-cycle control; the second is an example of closed-cycle control.

A servomechanism is a closed-cycle controller having the following characteristics: (i) the power to operate the controller, and the input signal to which the controller responds, are derived from separate sources; in other words, the load on the controller does not affect the signal itself; (ii) the sensitive element of the controller is actuated by the *difference* between the input signal and some function of the output signal.

The term *servomechanism* is derived from the Latin *servus*, a slave. A servomechanism is a device that tries to make the output slavishly follow the input.

Servomechanisms may be electrical, mechanical, hydraulic, or any combination. For brevity and concreteness, we will restrict ourselves to a selected series of electrical and mechanical combinations, and we will classify them according to the order of the linear differential equation giving the first approximation to their properties in the limit of very low frequency of variation of the input.

### The Simplest Example: A Zeroth-Order Servomechanism

Most college physics laboratories are supplied with 60-c/sec alternating current by the local power company, and then use some of this power to operate a motor generator to produce the direct current required by the laboratory. Usually the generator field is compound wound. The e.m.f. of the d.c. output can be varied by a rheostat in the shunt winding of the generator field. Usually the generator works well enough when the output e.m.f. is near the top of the range; but, usually, the performance is decidedly unsatisfactory for low-voltage work, as the e.m.f. then becomes a rapidly varying function of the output current.

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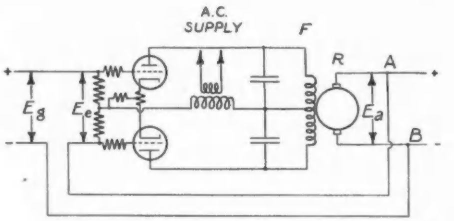


FIG. 1. A very simple servomechanism. It is an arrangement which assures that the output e.m.f., $E_a$, of the generator is quite independent of the speed of its rotor, of the load being supplied, and so forth. Here $E_g$ is the input signal, $F$ is the field of the generator, and $R$ is the rotor of the generator. For all practical purposes, provided the load is not sufficient to stall the driving motor (not shown in the figure) nor to require field currents that saturate the rectifier tubes, $E_a$ is a function only of the applied signal $E_g$. Of course, the points $A$ and $B$ can be anywhere desired in the circuit excited by the generator.

One remedy would be to eliminate the series windings, and to provide separate excitation for the shunt windings; in most cases, the output of a small rectifier would suffice. It would be still better to rewind the generator with a center-tapped field, and then to excite it in the manner shown in Fig. 1. Generators with fields wound in this manner are made by the General Electric Company, and sold under the trade name of Amplidynes. (The armatures of amplidyne generators are also of special construction.)

To see how this device works, let $E_g$ be a steady d.c. signal, such as the output of a battery of dry cells. Then, if the polarity is that shown in Fig. 1, the upper tube in the figure conducts much more than the lower tube, and the magnetic field produced by the upper half of the field winding is stronger than (and opposite to) the field produced by the lower half, the circuit being adjusted so that these two fields are equal in magnitude and opposite in direction when $E_g$ is zero. Then, if we are operating on the linear portion of the tubes' characteristics, and if such conditions as adequacy of the power supplies are met, we have

$$E_a = K_a E_e \tag{1}$$

$$\begin{aligned} &= K_a(E_g - E_a) \\ &= K_a E_g/(1+K_a), \end{aligned} \tag{2}$$

where $K_a$, the response (or "gain") of the amplifier and generator combined, is, for the frequencies in which we are interested, a scalar parameter. This parameter must be large, but it need not be a constant; indeed, it will, in general, be a function of the rotor speed, of the line voltage, of the applied load, and so forth. The difference $E_g - E_a [= E_e]$ is called the "error signal," or often, simply the "error," and $K_a$ is the response of the system to this error. But if $K_a$ be very large,

$$\lim_{K_a\to\infty} E_a = \lim_{K_a\to\infty} K_a E_g/(1+K_a) = E_g,$$

$$\lim_{K_a\to\infty} \mathrm{d}E_a/\mathrm{d}K_a = \lim_{K_a\to\infty} E_g/(1+K_a)^2 = 0.$$

Thus, $E_a$ is quite independent of fluctuations in $K_a$ if $K_a$ be large.

However, $E_g$ need not be a d.c. signal; if it be more convenient, it may be an a.c. signal, which *must* then be in phase with the plate voltage. The two plates will always go positive together, whereas the two grids will always have opposite polarity. Then, the tube whose plate and grid go positive together will conduct more than the other tube. If the phase of $E_g$ be reversed, the polarity of $E_a$ will likewise be reversed. The combination of amplifier and generator is therefore an example of a sense-detecting rectifier. Thus, for many applications, $E_g$ might be the output of a Variac, and $K_a$ would then be the ratio of $E_a$ to the amplitude of the voltage wave $E_g$.

Suppose, now, we wish the speed of an electric motor to be independent of variations in the power supply and in the load driven by the motor. For simplicity, let the motor be a separately excited d.c. motor, and let its armature be supplied by the output of a sense-detecting rectifier. Let a small, separately excited d.c. generator (permanent-magnet generators are frequently used for the purpose) be connected to the shaft of the motor, and let the output of the generator be connected to the input, as shown in Fig. 2. Let the speed of the motor's armature be denoted by $\Omega_m$, where

$$\Omega_m = K_m E_a, \tag{3}$$

and $K_m$, the response of the motor, is, for the time being, another scalar parameter, whose value will depend in part upon the load on the motor. Let the output of the velocity generator be

$$E_v = K_v \Omega_m, \tag{4}$$

where $K_v$, the response of the velocity generator, is a scalar constant. Then, combining Eqs. (1),

(3) and (4), we obtain

$$E_v = K_aK_mK_vE_e = K_aK_mK_v(E_g - E_v)$$
$$= K_aK_mK_vE_g/(1+K_aK_mK_v), \qquad (5)$$

and

$$\Omega_m = K_aK_mE_g/(1+K_aK_mK_v). \qquad (6)$$

Thus, $E_v$ remains closely equal to $E_g$ if the product $K_aK_mK_v$ is large compared to unity; therefore $\Omega_m$ will remain close to $E_g/K_v$, independently of fluctuations in $K_a$ and in $K_m$, and, therefore, independently of fluctuations in the load on the motor (provided, of course, that the load is not too great for the power supply, and so forth).

### A First-Order Servomechanism

Equation (6) applies to the final steady state when $E_g$ is a constant, but it would not apply when $E_g$ is a variable unless the inertia of the load driven by the motor were negligibly small compared to the resistances and drags overcome by it. If the inertia is not negligible, then $K_m$ is no longer a scalar parameter, and we must replace its reciprocal by the linear operator,

$$1/K_m = A_m\mathrm{d}/\mathrm{d}t + B_m,$$

where $A_m$ is a parameter proportional to the total inertia (armature, gear trains, and everything else) driven by the motor, and $B_m$ is a parameter proportional to the "viscous" drags and resistances overcome by the motor. Then Eq. (3) becomes

$$A_m(\mathrm{d}\Omega_m/\mathrm{d}t) + B_m\Omega_m = E_a. \qquad (7)$$

From Eq. (1) we then have

$$A_m\dot{E}_v + B_mE_v = K_vE_a = K_vK_a(E_g - E_v),$$

or

$$A_m\dot{E}_v + (B_m + K_vK_a)E_v = K_vK_aE_g; \qquad (8)$$

then, from Eq. (4),

$$A_m\mathrm{d}\Omega_m/\mathrm{d}t + (B_m + K_vK_a)\Omega_m = K_aE_g. \qquad (9)$$

The solutions to Eqs. (8) and (9) contain two terms—a transient term and a term depending upon the functional form of $E_g$. Thus, if $E_g$ be a suddenly applied constant, so that the motor is started suddenly from rest at the time $t=0$,

$$\Omega_m = \frac{K_aE_g}{B_m+K_vK_a}\left[1-\exp\left(-\frac{B_m+K_vK_a}{A_m}t\right)\right]. \qquad (10)$$

The quantity $A_m/(B_m+K_vK_a)$ is the time constant of the decaying transient. It is a function of the load as well as of the circuit parameters. After the transient has decayed, Eq. (10) becomes the same as Eq. (6).

If $\dot{E}_g$ be constant, so that $E_g = \dot{E}_g t$, and if the motor be started from rest when $t=0$, then

$$\Omega_m = \frac{K_a\dot{E}_g}{B_m+K_vK_a}\left[t - \frac{A_m}{B_m+K_vK_a}\right.$$
$$\left.\times\left(1-\exp\left(-\frac{B_m+K_vK_a}{A_m}t\right)\right)\right], \qquad (11)$$

and

$$\frac{\mathrm{d}\Omega_m}{\mathrm{d}t} = \frac{K_a\dot{E}_g}{B_m+K_vK_a}\left[1-\exp\left(-\frac{B_m+K_vK_a}{A_m}t\right)\right]. \qquad (12)$$

Thus, when $\dot{E}_g$ is constant, the acceleration of the motor shaft takes the role previously played by the velocity of the shaft when $E_g$ was constant.

If $E_g$ be a sinusoidal input, of frequency $\omega$, and of amplitude $|E_g|$, so that $E_g = |E_g|\sin\omega t$, where $\omega$ is very small—say, 0.5 $\sec^{-1}$ or less in many applications of interest—and if the motor be started from rest when $t=0$, then

$$\Omega_m = \frac{K_a|E_g|}{[(A_m\omega)^2+(B_m+K_vK_a)^2]^{\frac{1}{2}}}$$
$$\times[\sin(\omega t-\phi)+\sin\phi e^{-\omega t\cot\phi}], \qquad (13)$$

where $\tan\phi = A_m\omega/(B_m+K_vK_a)$. Thus, the amplitude of the oscillation of $\Omega_m$ and the phase lag of $\Omega_m$ behind $E_g$ are functions of the applied frequency $\omega$.

Since any input signal of practical interest can always be resolved into Fourier components, we

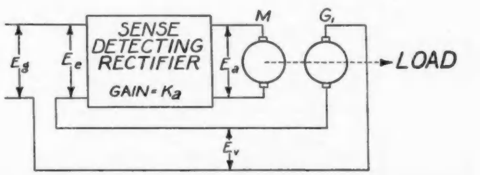


FIG. 2. A servomechanism for controlling the speed $\Omega_m$ of a separately excited d.c. motor, whose rotor is designated by $M$. Here $G_1$ is the rotor of a separately excited d.c. generator whose e.m.f., $E_v$, is proportional to $\Omega_m$; $\Omega_m$ is quite proportional to $E_g$ as $E_g$ is varied from zero to the highest value for which the controller is designed. If $E_g$ is reversed, $E_a$ and $\Omega_m$ are also reversed. Thus $\Omega_m$ can be varied accurately and continuously in any desired manner.

can always find the response of the system to an arbitrary input.

We now seek to reduce the phase lag $\phi$. One obvious way is to attempt to reduce $A_m$ or to increase $B_m+K_vK_a$. Now, in most applications, $A_m$ is not easily changed, whereas $B_m$ can always be increased by adding eddy-current disks to the motor shaft, or by other similar means. Such methods waste power, and present the problem of removing the heat produced. We therefore desire a method of reducing $\phi$ without greatly increasing $B_m$. The parameter $K_v$ cannot be altered without changing the ratio $\Omega_m/E_g$. But from the equations so far developed, there would appear to be no limitation upon $K_a$.

However, Eqs. (1), (3), (4) and (7) are but approximations; no physical amplifier, motor or generator are described by them exactly. The equations derived from them are therefore but approximations. The approximations are good enough for fairly large values of $K_a$, but fail as $K_a$ is still further increased. The next close approximations lead us to differential equations of higher order, and from the solutions to these equations we can see that there is a value of $K_a$ at which the system becomes a self-excited oscillator, and above which the system becomes unstable.

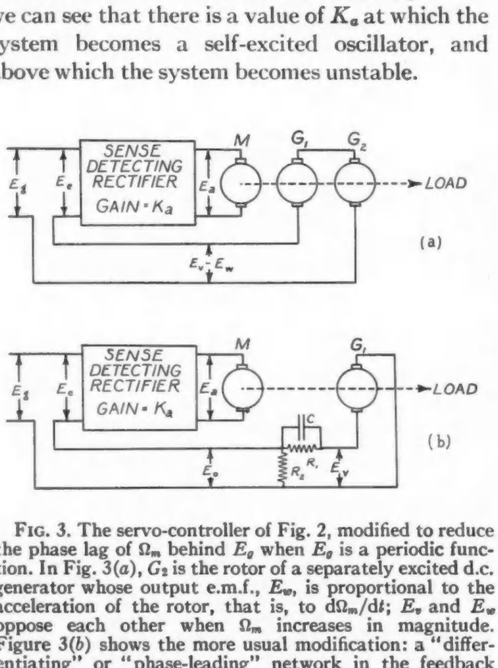


FIG. 3. The servo-controller of Fig. 2, modified to reduce the phase lag of $\Omega_m$ behind $E_g$ when $E_g$ is a periodic function. In Fig. 3($a$), $G_2$ is the rotor of a separately excited d.c. generator whose output e.m.f., $E_w$, is proportional to the acceleration of the rotor, that is, to $d\Omega_m/dt$; $E_v$ and $E_w$ oppose each other when $\Omega_m$ increases in magnitude. Figure 3($b$) shows the more usual modification: a "differentiating" or "phase-leading" network in the feedback loop. Much more complicated differentiating networks are also used.

Let us now consider two possible modifications of the system, as shown in Fig. 3. First, let us add to the load driven by the motor a d.c. generator whose output is proportional to the acceleration of its shaft. Of course, this will increase both $A_m$ and $B_m$. Let the output $E_w$ of this acceleration generator be given by

$$E_w = K_w d\Omega_m/dt. \tag{14}$$

Let the outputs of the velocity generator and of the acceleration generator be connected as shown in Fig. 3($a$). Then

$$E_e = E_g - E_v + E_w. \tag{15}$$

Combining Eqs. (4), (7), (14) and (15), we obtain

$$(A_m - K_aK_w)d\Omega_m/dt + (B_m + K_aK_v)\Omega_m = K_aE_g. \tag{16}$$

Thus, the first derivative term would vanish when $A_m = K_aK_w$, and the servomechanism would be changed to one of zeroth order.

However, it is impossible to lower the order of a servomechanism, for reasons quite similar to those that set the limit upon the value of $K_a$. Thus, it is impossible to cause the coefficient of $d\Omega_m/dt$ in Eq. (16) to vanish exactly by the addition of an acceleration generator.

The reduction of $A_m - K_aK_w$ to a very small quantity does not mean that the system would then respond to a suddenly applied $E_g$ with infinite acceleration. The acceleration can never be greater than the maximum obtainable from the available power supply. Similarly, the phase lag $\phi$ would be reduced only to the extent that the accelerations required can be produced by the power supply.

Also, if $E_g$ be a variable, it will always contain noise components of high frequency superimposed on the intended variations. Normally, the servo does not respond to this noise, but decreasing the magnitude of $A_m - K_aK_w$ enhances the response of the system to these unwanted high-frequency components.

Now, let us consider the insertion of a so-called "differentiating network" into the feedback loop. This is the customary approach to the problem. For the simple differentiating network shown in Fig. 3($b$), we will have

$$R_1R_2C\dot{E}_o + (R_1+R_2)E_o = R_1R_2C\dot{E}_v + R_1E_v, \tag{17}$$

and

$$E_e = E_g - E_0. \tag{18}$$

Combining Eqs. (1), (4), (7), (17) and (18), we obtain

$$R_1R_2CA_m\mathrm{d}^2\Omega_m/\mathrm{d}t^2+[(R_1+R_2)A_m+R_1R_2C(B_m+K_aK_v)]\mathrm{d}\Omega_m/\mathrm{d}t$$
$$+[(R_1+R_2)(B_m+K_vK_a)-R_2K_vK_a]\Omega_m=K_a[R_1R_2C\dot{E}_g+(R_1+R_2)E_g]. \quad (19)$$

The discriminant of the auxiliary equation is

$$[(R_1+R_2)A_m-R_1R_2C(B_m+K_vK_a)]^2+4R_1R_2^2CA_mK_vK_a,$$

and since this is always positive, there can be no oscillations in the transient response. Then, if $E_g$ be a constant, we will have, after the transients have decayed,

$$\Omega_m=K_a(R_1+R_2)E_g/[(R_1+R_2)(B_m+K_vK_a)-R_2K_vK_a]. \quad (20)$$

If $\dot{E}_g$ be a constant, then, after the decay of the transients,

$$\Omega_m=\frac{K_a\dot{E}_g}{(R_1+R_2)(B_m+K_aK_v)-R_2K_aK_v}\left[(R_1+R_2)t-\frac{(R_1+R_2)^2A_m+R_1R_2^2CK_vK_a}{(R_1+R_2)(B_v+K_vK_a-R_2K_vK_a)}\right]. \quad (21)$$

Thus, if $\Omega_m/E_g$ is to have the same magnitude (approximately) as in Eqs. (10) and (11), $K_v$ will have to be multiplied by $(R_1+R_2)/R_1$.

If $E_g=|E_g|\sin\omega t$, the solution to Eq. (19) would be easy to obtain, but long to write out. Perhaps the following observations might be more useful than the lengthy equation.

From Eq. (7) we see that, with or without the differentiating network, $\Omega_m$ lags behind $E_a$ by the phase angle whose tangent is $\omega A_m/B_m$. From the construction shown in Figs. 2 and 3(*b*), we see that $E_v$ lags behind $E_e$ by this same phase angle. Equation (17) shows that $E_o$ leads $E_v$ by the phase angle whose tangent is $\omega R_2^2C/[R_1+R_2+R_1(\omega R_2C)^2]$. Then if $E_o$ have the same amplitude in Fig. 3(*b*) as $E_v$ had in Fig. 2, $E_e$ will lag behind $E_g$ by a smaller angle in Fig. (3*b*) than in Fig. 2. Thus, the lag $\phi$ of $\Omega_m$ behind $E_g$ is diminished by the same amount.

### A Second-Order Servomechanism: The Zero-Displacement-Error Servo

The zero-displacement-error servo is commonly called by the briefer name *position servo*. In much of the literature, it is the first example discussed. A typical position servo is shown in Fig. 4. The motor is supposed to move an object (through suitable gear trains, rack and pinion, connecting rods, or other means) to some desired position, and hold it there, the desired position being indicated by the input signal $E_g$. The sliding contact of the potential divider is attached to one of the moving parts driven by the motor. The motor therefore runs until the output of the potential divider is equal and opposite to $E_g$. Thus, the function of the potential divider is to cause the motor to stop at the right place.

Position servos are essential components of automatic computers, fire-control apparatus, and so forth. For example, the simple position servo shown is used to interpose and maintain the proper angle between the optical axis of a telescope and the bore axes of the guns of an antiaircraft battery firing on a moving target.

Let the position of the motor shaft be denoted by $\Theta_m$, and let the output of the potential divider $E_p$ be given by

$$E_p=K_p\Theta_m, \quad (22)$$

where $K_p$ includes the excitation of the potential divider, the reduction of gear trains, and everything else that affects the ratio $E_p/\Theta_m$. Then Eq. (7) becomes

$$A_m\frac{\mathrm{d}^2\Theta_m}{\mathrm{d}t^2}+B_m\frac{\mathrm{d}\Theta_m}{\mathrm{d}t}=K_a(E_g-E_p),$$

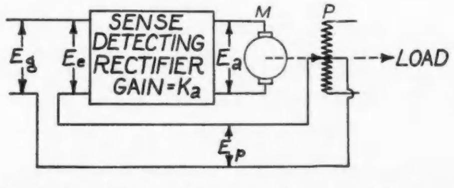


FIG. 4. A typical position servo. The motor $M$ moves the sliding contact of the potential divider $P$ until $E_p$ is equal and opposite to $E_g$.

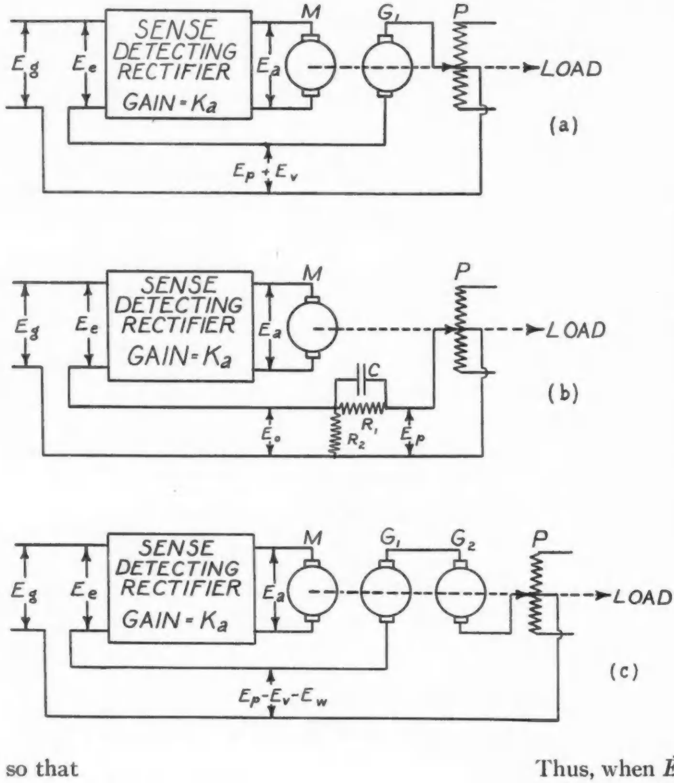


FIG. 5. The position servo of Fig. 4 modified to improve its response. In Figs. 5(*a*) and 5(*b*) are shown modifications to improve the transient response, that is, to eliminate oscillations if the original system be underdamped. In Fig. 5(*a*), $G_1$ is the rotor of a separately excited d.c. generator whose output e.m.f., $E_v$, is proportional to the speed $\Omega_m$ of the servo motor $M$, so that $E_v$ is also proportional to $\dot{E}_p$. In Fig. 5(*b*), a differentiating network had been placed in the feedback loop. Both schemes are in wide use. Figure 5(*c*) shows a method for reducing the phase lag of $E_p$ behind $E_g$ when $E_g$ is a periodic function; $G_2$ is the rotor of a separately excited d.c. generator whose output e.m.f., $E_w$, is proportional to the acceleration of the servo motor.

so that

$$A_m\frac{d^2\Theta_m}{dt^2}+B_m\frac{d\Theta_m}{dt}+K_pK_a\Theta_m=K_aE_g. \qquad (23)$$

The transient terms in the solutions to Eq. (23) will depend upon the nature of the discriminant $B_m^2-4A_mK_pK_a$. If the discriminant be negative, the system is underdamped, and the transient response will contain an oscillation of frequency $(4A_mK_pK_a-B_m^2)^{\frac{1}{2}}$. If the discriminant be zero, the system is critically damped. If it be positive, the system is overdamped. Of course, the system does not obey the equations exactly anyway, and, in any event, critical damping is hard to produce and still harder to maintain. In many applications, the best compromise is to have the system a bit underdamped, so that there is but one observable overshoot; in other applications, it is necessary that the system be at least a bit overdamped.

If $E_g$ be constant, we will have, after the transients have decayed,

$$\Theta_m=E_g/K_p. \qquad (24)$$

If $E_g=\dot{E}_gt$, where $\dot{E}_g$ is constant, we will have, also after the transients have decayed,

$$\Theta_m=\dot{E}_g(t-B_m/K_pK_a)/K_p. \qquad (25)$$

Thus, when $\dot{E}_g$ is constant, $\Theta_m$ lags behind by an amount sufficient to produce the error signal required to cause the motor to run at such a speed that $\dot{E}_p=\dot{E}_g$. This lag is called a "velocity error."

If $E_g=|E_g|\sin\omega t$, the steady-state solution will be

$$\Theta_m=K_a|E_g|\sin(\omega t-\phi)/[(K_pK_a-\omega^2A_m)^2+(\omega B_m)^2]^{\frac{1}{2}}, \qquad (26)$$

where $\tan\phi=\omega B_m/(K_pK_a-\omega^2A_m)$. Thus, even if there were no inertia in the system at all, there would still be a finite phase lag $\phi$, and the amplitude of $\Theta_m$ would still be a function of the applied frequency $\omega$.

However, in many cases, $A_m$ is quite large, and it is necessary to devise means to increase the effective damping so as to reduce or even eliminate the sinusoidal terms in the transient response, without unduly increasing $B_m$. One obvious suggestion is to reduce $K_a$, the most easily adjusted parameter in the circuit. But the sensitivity of the controller is proportional to $K_a$;

if $K_a$ be too small, the minimum error to which the system will respond becomes too large.

Therefore, let us consider some very simple modifications of the controller.

The simplest modification is that shown in Fig. 5($a$). A velocity generator is attached to the armature shaft of the servo motor, and its output is placed in series with $E_p$. Then, if the output of the generator be given by

$$E_v = K_v\Omega_m = K_v \mathrm{d}\Theta_m/\mathrm{d}t \tag{4}$$

as before, we have

$$A_m\frac{\mathrm{d}^2\Theta_m}{\mathrm{d}t^2}+B_m\frac{\mathrm{d}\Theta_m}{\mathrm{d}t}=K_aE_e=K_a(E_g-E_p-E_v),$$

and therefore

$$A_m\frac{\mathrm{d}^2\Theta_m}{\mathrm{d}t^2}+(B_m+K_vK_a)\frac{\mathrm{d}\Theta_m}{\mathrm{d}t}+K_pK_a\Theta_m=K_aE_g. \tag{27}$$

The effect of adding the velocity generator can be seen by comparing Eq. (27) with Eq. (23).

In many applications, however, one uses a "differentiating network" instead of the velocity generator. If we use the same network as that shown in Fig. 3($b$), we will obtain the third-order equation

$$\begin{aligned}R_1R_2CA_m&\frac{\mathrm{d}^3\Theta_m}{\mathrm{d}t^3}\\&+[(R_1+R_2)A_m+R_1R_2CB_m]\frac{\mathrm{d}^2\Theta_m}{\mathrm{d}t^2}\\&+[(R_1+R_2)B_m+R_1R_2CK_pK_a]\frac{\mathrm{d}\Theta_m}{\mathrm{d}t}\\&+R_1K_pK_a\Theta_m\\&\qquad=K_a[R_1R_2C\dot{E}_g+(R_1+R_2)E_g].\end{aligned} \tag{28}$$

We will say more about third-order differential equations in the next section. Meanwhile, we will note that the output of the velocity generator is proportional to the first time derivative of the output signal $E_p$, and that therefore

$$E_p+E_v=E_p+K_v\dot{E}_p/K_p.$$

The right-hand member of this equation is of the same form as that of the equation for the network; they will be alike if $R_2C=K_v/K_p$. The two left-hand members will be comparable for those frequencies for which $\omega R_2C$ is small. Thus, if the free transient response (without the differentiating network) contains frequencies small compared to $1/R_2C$, the effect of the network is comparable to that of the velocity generator.

We now come to the problem of reducing the phase lag $\phi$, if possible, in Eq. (26). To do this, we have to reduce $B_m+K_vK_a$, as the phase lag for zero inertia is given by

$$\tan\phi=\omega(B_m+K_vK_a)/K_pK_a,$$

and, at the same time, we wish to avoid the presence of oscillations in the transient response. If we were to insert an acceleration generator, as shown in Fig. 5($c$), we would obtain

$$(A_m-K_aK_w)\frac{\mathrm{d}^2\Theta_m}{\mathrm{d}t^2}+(B_m-K_aK_v)\frac{\mathrm{d}\Theta_m}{\mathrm{d}t}+K_aK_p\Theta_m=K_aE_g. \tag{29}$$

As we have mentioned before, no physical apparatus will ever satisfy such an equation exactly, and the approximation represented by these equations fails when we encounter the small difference between two large quantities in the coefficients.

### A Third-Order Servomechanism

For delicate control problems, modern hydraulic motors have an important advantage over electric motors: their rotors have but a fraction of the inertia of the armatures of electric motors of comparable power output. The moment of inertia of a powerful electric motor is very large. Then, if the motor drives its load through a large gear reduction, the angular momentum of the armature may be so large that it is the chief contributor to $A_m$ in Eq. (7). Modern hydraulic motors, on the other hand, are of satisfactory efficiency, and contribute but little to the total inertia being driven.

Figure 6 represents a possible servomechanism for controlling the speed of a powerful hydraulic motor. It could just as well represent a servo for controlling the speed of the automobile used as an example in the opening discussion of the difference between open- and closed-cycle controllers. The input signal $E_g$ would then be determined by the position of the operating lever or pedal. Here, the small,

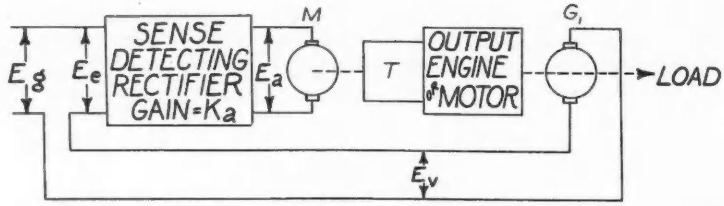


FIG. 6. A servo for controlling a steam engine, an internal combustion engine, or a hydraulic motor, whose speed $\Omega_n$ is determined by the position of the throttle or valve $T$; $G_1$ is a separately excited d.c. generator whose output e.m.f., $E_v$, is proportional to $\Omega_n$. Should any change in the load, or in any operating condition, affect $\Omega_n$, the resulting change in $E_v$ will cause the servo motor $M$ to readjust the throttle or valve until the desired speed is again obtained.

light, servo motor merely controls the position of the throttle that regulates the speed of the heavier output motor. If the output load change, the speed $\Omega_n$ of the output motor will change, causing $E_v$, and then $E_e$ and $E_a$, to change. The change in $E_a$ will cause the servo motor to run until the valve or throttle is readjusted to cause the output motor to run at the desired speed. In some cases, the servo motor is a rectilinear device actuated directly by the sense-detecting rectifier, and having inertia small compared to that of the armature of an electric motor.

In any case, the response of the servo motor is still given by Eq. (7), which we will now rewrite as

$$A_m\frac{d^2\Theta_m}{dt^2}+B_m\frac{d\Theta_m}{dt}=E_a=K_a(E_g-E_v). \quad (30)$$

The speed of the output motor is denoted by $\Omega_n$, where

$$A_n d\Omega_n/dt+B_n\Omega_n=\Theta_m, \quad (31)$$

$A_n$ represents the total inertia driven by the output motor, and $B_n$ represents the total drags overcome by it. For the velocity generator attached to its shaft, we will have

$$E_v=K_v\Omega_n. \quad (32)$$

Combining Eqs. (30), (31) and (32), we will have

$$A_mA_n\frac{d^3\Omega_n}{dt^3}+(A_mB_n+B_mA_n)\frac{d^2\Omega_n}{dt^2}$$

$$+B_mB_n\frac{d\Omega_n}{dt}+K_aK_v\Omega_n=K_aE_g. \quad (33)$$

For brevity, let us write this as

$$M\frac{d^3\Omega_n}{dt^3}+N\frac{d^2\Omega_n}{dt^2}+P\frac{d\Omega_n}{dt}+Q\Omega_n=K_aE_g. \quad (33)$$

To find the transient terms in the response of this system, we have to solve the auxiliary cubic equation, whose discriminant is

$$9MQ(2NP-3MQ)-4(MP^3+N^3Q)+N^2P^2.$$

A cubic equation having real coefficients has three roots: one root will always be real; the others will be real, or else they will be complex conjugates. Thus, the transient response will consist of one pure exponential term, plus two other exponential terms which may or may not contain sinusoidal components. If the servomechanism is to be stable, the real parts of these exponentials must be negative.

The expressions for the roots of a cubic equation are difficult to use. Graphical methods for locating the roots and for the determining the stability of the servomechanism have been prepared by Liu and by Evans at the Servomechanisms Laboratory at the Massachusetts Institute of Technology.[1] The stability of third- and fourth-order systems is discussed by von Karman and Biot.[2] But, without going through the complete analysis, we can see by inspection of Eq. (33) that when $P/M=Q/N=\omega_0^2$, the system becomes a self-excited oscillator of frequency $\omega_0$. When $Q/N$ is greater than $P/M$, the oscillations increase until saturation of the amplifier or other physical limitation prevents further increase in the amplitude. When $Q/N$ is less than $P/M$, any oscillation in the transient response decays exponentially.

After the transients have decayed, the response to a constant input will be

$$\Omega_n=E_g/K_v. \quad (34)$$

If the input have a constant first derivative, the steady-state response will be

$$\Omega_n=\dot{E}_g(t-B_mB_n/K_aK_v)/K_v. \quad (35)$$

[1] Y. J. Liu, *Servomechanisms, charts for verifying their stability* (M. I. T., 1941); L. W. Evans, *Solutions of cubic equations, and cubic charts* (M. I. T., 1943). See also Jahnke and Emde, *Tables of functions* (Teubner, 1933; Dover, 1943), pp. 20–30.

[2] Th. von Karman and M. A. Biot, *Mathematical methods for physics and engineering* (McGraw-Hill, 1940), pp. 242–247.

Thus, if $K_a$ be large, $\Omega_n$ is seen to be quite insensitive to variations in $A_n$ or in $B_n$.

If $E_g = |E_g|\sin\omega t$, the steady-state response will be

$$\Omega_n = K_a|E_g|\sin(\omega t-\phi)/ [\omega^2(P-\omega^2M)^2+(Q-\omega^2N)^2]^{\frac{1}{2}}, \quad (36)$$

where $\tan\phi=\omega(P-\omega^2M)/(Q-\omega^2N)$. Thus, $\tan\phi$ may be either positive or negative. We now want to examine this condition, and also determine the significance of the indeterminate case for $\tan\phi$.

Let us now open the feed-back loop connecting $E_v$ and $E_g$, and find the response of the resulting system to $E_e$ when $E_e=|E_e|\sin\omega t$. The response of the servo motor will be the steady-state solution of the equation, $A_m\mathrm{d}^2\Theta_m/\mathrm{d}t^2+B_m\mathrm{d}\Theta_m/\mathrm{d}t = K_aE_e$; that is,

$$\Theta_m=\frac{K_a|E_e|\sin(\omega t-\frac{1}{2}\pi-\phi_m)}{\omega[(\omega A_m)^2+B_m^2]^{\frac{1}{2}}},$$

where $\tan\phi_m=\omega A_m/B_m$. Thus, $\Theta_m$ will lag behind $E_e$ by at least $\pi/2$. If we substitute this expression for $\Theta_m$ into Eq. (31), we obtain

$$\Omega_n=\frac{K_a|E_e|\sin(\omega t-\frac{1}{2}\pi-\phi_m-\phi_n)}{\omega[[(\omega A_m)^2+B_m^2][(\omega A_n)^2+B_n^2]]^{\frac{1}{2}}},$$

and if we multiply both sides of this equation by $K_v$, we will have

$$E_v=K_v\Omega_n=\frac{Q|E_e|\sin(\omega t-\frac{1}{2}\pi-\phi_m-\phi_n)}{\omega[(\omega N)^2+(P-\omega^2M)^2]^{\frac{1}{2}}}, \quad (37)$$

where $\tan\phi_n=\omega A_n/B_n$, and $\tan(\phi_m+\phi_n)=\omega N/(P-\omega^2M)$. In what follows. it will be more convenient to consider $E_v$ rather than $\Omega_n$. Thus, $E_v$ lags behind $E_e$ by at least $\pi/2$.

Now, let us see what happens in the original system with the feed-back loop closed. If the lag of $E_v$ behind $E_e$ be less than $\pi$, $E_g$ will lie between $E_e$ and $E_v$, and since $E_e$ will be small compared to $E_g$, $E_v$ will lag behind $E_g$ by a relatively small angle; if $E_v$ lag behind $E_e$ by more than $\pi$, $E_v$ will lead $E_g$.

Consider now the effects produced when the lag of $E_v$ behind $E_e$ is $\pi$. This occurs when $\phi_m+\phi_n=\pi/2$, or when $P=\omega^2M$. Suppose that $E_g$ is kept at zero, and that the system is subjected to a sudden disturbance imparting some speed to the output motor. Since $E_g$ is zero, we will have $E_e=-E_v$, and, of course, the effect of $E_e$ should be to drive the output motor to rest. But if $E_e$ contain a component whose frequency is given by $\omega_0^2=P/M$, the resulting component of $E_v$ will be in phase with that component of $E_e$. If, in addition, $Q/\omega_0^2N=1$, we see from Eq. (37) that this component of $E_v$ will have the same magnitude as that of $E_e$, or, in other words, this component of $E_v$ and of $E_e$ will propagate itself around the loop undiminished; we then have the self-excited oscillator, and the indeterminate case of $\tan\phi$.

If $Q/\omega_0^2N$ be greater than unity, $E_v$ and $E_e$ will grow in magnitude until further increase is prevented by the physical limitations of the apparatus; if it be less than unity, the resulting disturbance will decay exponentially.

We now seek to make $\omega_0$ as large as possible, or, at least, to make it higher than the frequencies of the components of any likely disturbance of the system; we also wish $Q/\omega_0^2N$ to be small. One way is to make $\phi_m+\phi_n$ as small as possible at the frequencies at which the controller will have to operate. In most applications, $\phi_n$ is relatively fixed, but $\phi_m$ can be decreased by substituting for the servo motor just used, one whose armature is held in the zero position by a suitable spring. If the restoring effect of this spring be denoted by $C_m$, Eq. (30) becomes

$$A_m\frac{\mathrm{d}^2\Theta_m}{\mathrm{d}t^2}+B_m\frac{\mathrm{d}\Theta_m}{\mathrm{d}t}+C_m\Theta_m=E_a=K_a(E_g-E_v), \quad (38)$$

and the resulting changes in Eqs. (33) to (36) are easy to determine. If $E_e=|E_e|\sin\omega t$, the response of this servo motor will be

$$\Theta_m=\frac{K_a|E_e|\sin(\omega t-\phi_m')}{[(\omega^2A_m-C_m)^2+(\omega B_m)^2]^{\frac{1}{2}}},$$

where $\tan\phi_m'=\omega B_m/(C_m-\omega^2A_m)$; and since $C_m$ can be quite large, $\phi_m'$ can be quite small. It will facilitate comparison with the original system to rewrite this as

$$\Theta_m=\frac{K_a|E_e|\sin(\omega t-\frac{1}{2}\pi-\phi_m)}{[(\omega^2A_m-C_m)^2+(\omega B_m)^2]^{\frac{1}{2}}}, \quad (39)$$

where $\tan\phi_m=(\omega^2A_m-C_m)/\omega B_m$, and $\phi_m$ may be either positive or negative. Then, for the output

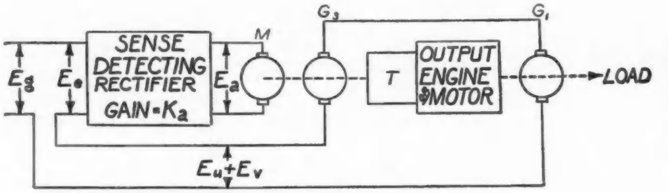


FIG. 7. The controller of Fig. 6, modified so as to reduce the phase lag of $\Omega_n$ behind $E_g$ when $E_g$ is a periodic function; $G_1$ and $G_3$ are the rotors of d.c. generators, as before. The output e.m.f., $E_u$, of $G_3$ is proportional to the speed $\Omega_m$ of the servo motor $M$. The output of $G_1$ is the same as in Fig. 6.

signal, we will have

$$E_v=\frac{K_aK_v|E_e|\sin(\omega t-\frac{1}{2}\pi-\phi_m-\phi_n)}{[(\omega^2N-B_nC_m)^2+\omega^2(P-\omega^2M)^2]^{\frac{1}{2}}},\qquad(40)$$

where $\tan(\phi_m+\phi_n)=(\omega^2N-B_nC_m)/\omega(P-\omega^2M)$. Thus, $\phi_m+\phi_n$ is now less that it was in Eq. (37).

The phase angles can be still further reduced, if desired, by attaching a velocity generator to the shaft of the servo motor, as shown in Fig. 7. If the response of this velocity generator be given by

$$E_u=K_u\mathrm{d}\Theta_m/\mathrm{d}t,\qquad(41)$$

then Eq. (30) becomes

$$A_m\frac{\mathrm{d}^2\Theta_m}{\mathrm{d}t^2}+B_m\frac{\mathrm{d}\Theta_m}{\mathrm{d}t}=K_aE_e=K_a(E_g-E_u-E_v),$$

or

$$A_m\frac{\mathrm{d}^2\Theta_m}{\mathrm{d}t^2}+(B_m+K_uK_a)\frac{\mathrm{d}\Theta_m}{\mathrm{d}t}=K_a(E_g-E_v),\qquad(42)$$

and the resulting changes in Eqs. (33) to (40) are easily written.

### A Fourth-Order Servomechanism: The Zero-Velocity-Error Servo

There are applications for which it is desirable to eliminate the constant terms in the right-hand members of Eqs. (11), (21), (25) and (35). These constant terms represent steady-state lags of the output behind an input whose first time derivative is constant for some specified interval; they are often called "velocity errors." They are directly proportional to the "velocity" of the input signal and, in many cases, at least, are quite independent of the inertias of the system. For concreteness and brevity we will confine ourselves to the elimination of the velocity error in Eq. (25).

To eliminate this error, we need a device that will cause the position servo motor to continue to run at a speed proportional to the "velocity" of the input after the error voltage $E_e$ has vanished. The simplest such device is shown in Fig. 8($a$). To simplify the discussion, let us assume initially that the inertias of the two motors are negligible, so that

$$\mathrm{d}\Theta_m/\mathrm{d}t=K_mE_a=K_mK_a(E_g-E_q),$$

and

$$\mathrm{d}\Theta_n/\mathrm{d}t=K_nE_b=K_nK_bE_p.$$

Then

$$E_p=K_p\Theta_m,$$

and

$$E_q=K_q\Theta_n,$$

where $K_m$ and $K_n$ have the same meaning as the $K_m$ in Eq. (3), and $K_p$ and $K_q$ have the same meaning as the $K_p$ in Eq. (22). Combining these four equations, we have

$$\frac{\mathrm{d}^2\Theta_n}{\mathrm{d}t^2}+K_aK_bK_mK_nK_pK_q\Theta_n=K_aK_bK_mK_nK_pE_g.\qquad(43)$$

From Eq. (43) we see that the device is a self-excited oscillator whose natural frequency $\omega_0$ is given by $\omega_0^2=K_aK_bK_mK_nK_pK_q$. The first servo motor will run until $E_e$ is zero. At the instant that it stops, the instantaneous value of $E_q$ will be the same as that of $E_g$. Also, at that instant, $\dot{E}_q$ will become constant, since the speed of the second servo motor is now a constant determined by the position at which the output arm of the first potential divider stopped. But, also, at that instant, $E_q$ overtook $E_g$, so that $\dot{E}_q$ could not have been equal to $\dot{E}_g$. Therefore, $E_e$ will not remain zero, but will change sign and become finite, causing the first servo motor to reverse. Thus, the system will oscillate, as shown by Eq. (43).

To quench these oscillations, let us modify the controller as shown in Fig. 8($b$). Then, for the response of the second servo motor, we will have

$$\mathrm{d}\Theta_n/\mathrm{d}t=K_nE_b=K_nK_b(E_p+E_c)$$
$$=K_nK_b[E_p+K_c(E_g-E_q)],$$

and, therefore,

$$\frac{d^2\Theta_n}{dt^2}+K_bK_cK_nK_q\frac{d\Theta_n}{dt}+K_aK_bK_mK_nK_pK_q\Theta_n=K_bK_cK_n\dot{E}_g+K_aK_bK_mK_nK_pE_g. \quad (44)$$

From Eq. (44), we can see that there will be no oscillations in the transient response of this system if $K_c^2>4K_aK_mK_p/K_bK_nK_q$. Thus, if the two parts of the system be alike (not a likely construction) this condition becomes $K_c>2$.

Let us now consider a physical servomechanism the moments of inertia of whose armatures are finite. We must now replace the scalar parameters $K_m$ and $K_n$ by the linear operators,

$$1/K_m=A_m d/dt+B_m,$$
$$1/K_n=A_n d/dt+B_n,$$

and the resulting equation for the complete system is

$$A_mA_n\frac{d^4\Theta_n}{dt^4}+(A_mB_n+B_mA_n)\frac{d^3\Theta_n}{dt^3}+(A_mK_bK_cK_q+B_mB_n)\frac{d^2\Theta_n}{dt^2}+B_mK_bK_cK_q\frac{d\Theta_n}{dt}+K_aK_bK_pK_q\Theta_n$$
$$=K_b\left[K_c\left(A_m\frac{d^2E_g}{dt^2}+B_m\dot{E}_g\right)+K_aK_pE_g\right]; \quad (45)$$

if we multiply both members of this equation by $K_q$, we can write it in the somewhat more compact form,

$$M\frac{d^4E_q}{dt^4}+N\frac{d^3E_q}{dt^3}+(P+Q)\frac{d^2E_q}{dt^2}+R\dot{E}_q+SE_q=Q\frac{d^2E_g}{dt^2}+R\dot{E}_g+SE_g. \quad (45)$$

To find the transient response of this system, we have to find the roots of the auxiliary quartic equation. A quartic equation with real coefficients has two pairs of roots: either pair may be real, or they may be complex conjugates, independently of the nature of the other pair, except that in the case of an equation all of whose coefficients are positive, only one pair may be pure imaginary. For the system to be stable, the real parts of all four roots must be negative; for the transients to be free of oscillations, the four roots must be real.

If $N(P+Q)R=MR^2+N^2S$, the system is a self-excited oscillator whose natural frequency $\omega_0$ is given by $\omega_0^2=R/N$; if $N(P+Q)R$ be larger than $MR^2+N^2S$, the system is stable, and the oscillations will decay exponentially.

If $E_g$ be constant, the solution of Eq. (45) becomes, after the decay of the transients,

$$\Theta_n=E_g/K_q. \quad (46)$$

If $\dot{E}_g$ be constant, the steady-state solution will be

$$\Theta_n=\dot{E}_gt/K_q=E_g/K_q. \quad (47)$$

If $E_g=|E_g|\sin\omega t$, we obtain rather lengthy

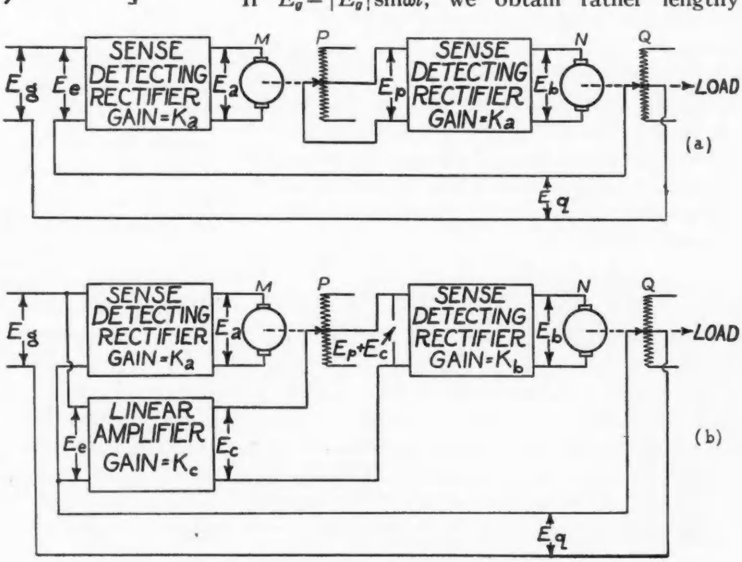


FIG. 8. Two steps in the construction of a zero-velocity-error position servo; $M$ and $N$ are the rotors of separately excited d.c. motors, and $P$ and $Q$ are potential dividers. The simple construction of two amplifiers and motors in cascade, as shown in Fig. 8(*a*), is unstable. The construction shown in Fig. 8(*b*) can always be made stable. Of course, modifications described for other servos in the text can be applied for this one, also.

expressions:

$$\Theta_n = \frac{|E_g|[(S-\omega^2Q)^2+(\omega R)^2]^{\frac{1}{2}} \sin(\omega t-\phi)}{K_q[[\omega^4M-\omega^2(P+Q)+S]^2+\omega^2(R-\omega^2N)^2]^{\frac{1}{2}}}, \tag{48}$$

where

$$\tan\phi = \frac{\omega[R[\omega^4M-\omega^2(P+Q)+S]-[R-\omega^2N][S-\omega^2Q]]}{[[\omega^4M-\omega^2(P+Q)+S][S-\omega^2Q]+\omega^2R[R-\omega^2N]]}.$$

The interpretation of these expressions for $\Theta_n$ and $\phi$ is facilitated by following a procedure similar to that used for examining Eq. (36): we consider the response of each part, both with the feedback loop opened, and with it left intact.

## Servomechanisms of Higher Order: Nyquist's Criterion

More complicated servos, as well as the more accurate description of the simpler ones, lead to differential equations of higher and higher order. The methods for determining the nature of the roots of their auxiliary equations are well known, but, all in all, the handling of such equations is quite tedious. However, one can always determine the stability of any servomechanism by the method proposed by Nyquist,[3] which we used for examining the properties of the third-order servomechanism above. First, we open the feedback loop, and find the amplitude and phase of the response of the resulting open-cycle system as functions of the frequency of the error signal considered as the input. We then find the frequency for which the phase lag is $\pi$: if the accompanying amplitude be less than that of the error signal used as the input, the system is stable; if the relative amplitudes be unity, the system is an oscillator; if the amplitude of the output be greater than that of the input, the system is unstable.

## Some Applications

*Stabilizers.*—In Figs. (2) and (6), the output signal was provided by a velocity generator whose output is proportional to the speed of its rotor with respect to its own stator. Suppose, now, we replace the velocity generator by a device whose output is proportional to its angular velocity in inertial (Galilean) space. We then have a stabilizer. Thus, the navigator aboard the ship mentioned in the opening discussion can use a stabilizer in which the input signal is kept at zero once the sextant has been placed in the desired position; the antiaircraft battery needs a stabilizer in which the input can be easily adjusted to make the guns follow the target.

*Automatic computers.*—Servomechanisms are the chief components of a variety of automatic computers, including the well-known differential analyzers. Perhaps an easily understood computer is the following oversimplified lead computer for the antiaircraft battery we have mentioned before.

Suppose the target is flying overhead in a straight line, and let the angular velocity of the line of sight from battery to target be $\omega$. Then, in order for the battery to be able to strike the target, it is necessary that the guns point to the "future position" of the target. Suppose that the angle $\alpha$ between the line of sight to present position and the line of sight to the future position is given with sufficient accuracy for the purpose by $\omega t_f$, where $t_f$ is the time of flight of the projectile from the gun to the present position of the target. Let the input to the first servomechanism in Fig. 9 be proportional to the distance from the gun to the present position of the target. Let this servo control the position of the sliding contact of a second potential divider whose windings are so spaced that they represent the relation between distance of travel and time of travel of the projectiles being used. The output of this potential divider will therefore be proportional to the time of flight of the projectiles from gun to target. Let the excitation of this potential divider be proportional to the angular velocity of the line of sight—in other words, let it be the output of a velocity generator attached to the trunnion of a telescope being made to follow the target. The

[3] H. Nyquist, *Bell Syst. Tech. J.* **11**, 126 (1932).

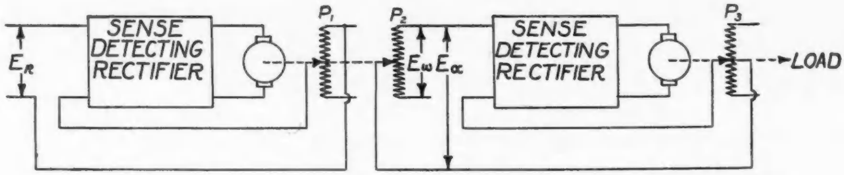


FIG. 9. An example of an automatic computer. The one shown is a device for computing and maintaining the desired lead angle $\alpha$ between the bores of the guns and the optical axis of a telescope mounted on the same trunnion axis with the guns; $E_R$ is a potential difference proportional to the distance to the target, obtainable either by radar or by optical means; $E_\omega$ is the output of a generator attached to the telescope following the target and is therefore proportional to the angular velocity of the target with respect to the observer. The position of the sliding contact on the potential divider $P_2$ is proportional to the range to the target, but the resulting output is proportional to the time of flight of the projectile over this range. Thus, $E_\alpha$ is a potential difference proportional to the desired lead angle $\alpha$.

output of this potential divider will now be proportional to $\omega t_f$, and is used as the input to the second servo, which merely maintains the proper lead angle between the axis of the telescope and the axis of the gun.

Of course, no real fire-control apparatus could be as simple as this. For one thing, it would be impossible to wind the second potential divider so as to reproduce the relation between distance and time of flight under all firing conditions.

*Differentiators.*—Many devices for obtaining the first time derivative of an arbitrary input have been developed during the past few years. Most of them were for inputs containing components of relatively high frequency, but there are problems for which it is desirable to be able to obtain derivatives of fairly long period. Also, many differentiators respond with greater amplitude to the higher-frequency components of the input than to the lower-frequency components, even differentiating the noise better than the signal. In any event, there will always be a phase lag between the true derivative of the signal and the response of the differentiator.

The error signal $E_e$ in Figs. (4) to (7) may be used as a representation of the first time derivative of the input signal $E_g$. If for the position servo of Fig. (4), we solve Eqs. (1), (7) and (22) for $E_e$ instead of for $E_p$, we obtain

$$A_m d^2E_e/dt^2 + B_m\dot{E}_e + K_aK_pE_e = A_m d^2E_g/dt^2 + B_m\dot{E}_g. \quad (49)$$

If $\dot{E}_g$ be constant, the steady-state solution is

$$E_e = B_m\dot{E}_g/K_aK_p. \quad (50)$$

If $E_g = |E_g|\sin\omega t$, so that $\dot{E}_g = |\dot{E}_g|\cos\omega t$, the steady-state solution is

$$E_e = \frac{|\dot{E}_g|[(\omega A_m)^2 + B_m^2]^{\frac{1}{2}}\cos(\omega t - \phi)}{[(K_aK_p - \omega^2A_m)^2 + (\omega B_m)^2]^{\frac{1}{2}}}, \quad (51)$$

where

$$\tan\phi = \omega[A_m(K_aK_p - \omega^2A_m) + B_m^2]/B_mK_aK_p.$$

Thus, the amplitude and phase of the response vary with the applied frequency, as they must in all differentiators, but here the response to the higher-frequency components is not enhanced.

Of course, $E_a$ could have been used instead of $E_e$. Further, the output $E_v$ of the velocity generator in Fig. 5($a$) can also be used as a representation of the derivative, but it should be obvious that $E_v$ is not as good for the purpose as $E_e$ or $E_a$, since $E_v$ must lag behind $E_a$ by the phase angle whose tangent is $\omega A_m/B_m$, and its amplitude must fall off with the frequency. Nevertheless, there is a current device in which $E_v$ is used instead of $E_a$ or $E_e$ to obtain the derivative of $E_g$.

Suppose that the input is an a.c. signal whose amplitude is made to vary in some arbitrary manner. Then $E_a$ will be a signal that gives the derivative of the amplitude $E_g$ of the a.c. voltage wave. Similarly, the error signal in Fig. 8($b$) can be used to give the second derivative of $E_g$.

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### Interference Films on Tungsten

A tungsten disk is made the anode in an electrolytic cell containing $0.06N$ NaOH; the cathode is an iron plate. When a potential difference of 100 to 200 v is applied through a series resistor, a thin film of oxide of tungsten forms on the disk, the thickness of which increases with time. When the potential difference is removed, the film dissolves. If the film is viewed in a strong light, it exhibits the interference color characteristic of its thickness; since the latter changes, the color also varies. The color may be made to cease changing by proper adjustment of the series resistor. If the disk is washed with water as it is removed from the cell, the film remains and is quite stable.—I. H. Parsons, *J. Chem. Ed.* **24**, 483 (1947).

### Combined Chemistry-Physics Course

At Bard College an integrated course in chemistry and physics has been developed. The first semester's work is integrated by the concept of the atomic-molecular theory of matter; it includes study of Newton's laws of motion, the basic laws of mass relations in chemical reactions, thermal expansion of solids and liquids, the laws of gases, and sufficient mathematics for tool purposes. In the second semester the work is approximately that of a course in "modern physics," together with direct-current electricity, leading to a study of chemical equilibrium.

The third semester is devoted to a systematic study of the important families of chemical elements and elementary qualitative analysis. The fourth semester includes surface phenomena, elastic moduli, hydrodynamics, rotary and vibratory motion, acoustics, geometric optics, magnetism, and alternating-current electricity.

The course as planned avoids the usual overlapping between chemistry and physics in certain topics and brings out clearly the integration of the two subjects; however, the first two semesters are somewhat theoretical, and some of the material usually covered in the first year of college chemistry is postponed to the end of the second year.—E. C. Fuller, *J. Chem. Ed.* **24**, 380–81 (1947).

# Demonstration Experiments with Pendulums

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## Straight Rods

A COMMON meter stick serves very well for a uniform thin rod. If it has metal binding at the ends a small hole may be drilled through this binding so that, when the stick is hung on a horizontal nail, the axis $A$ (Fig. 1) is (nearly enough) at one end. If $l_e$ is the length of the equivalent simple pendulum and $h$ is the distance from the axis to the center of mass, in general $l_e = I_A/Mh$, and in this case, approximately, $l_e = 2L/3 = 66.7$ cm. Then if a small lead ball is hung by a thread from the same axis it must be very nearly 66.7 cm from the axis in order to swing in unison with the rod.

Now it is convenient to have a short brass or iron cylinder with a short screw in one face that may be inserted in a threaded hole in a similar cylinder, so that the two form one cylinder of mass 1 kg or so when they are fastened together. These may be separated a little and clamped anywhere along the suspended stick. It will be found, of course, that when the cylinder is clamped at the center of oscillation $O$ (that is, at 66.7 cm from $A$), the period of vibration $T$ is unchanged. But if the cylinder is clamped at the center of mass $C$ of the stick, 50 cm from $A$, as shown in Fig. 1($c$), the stick and the simple pendulum of length 66.7 cm do not keep step at all.

Two more meter sticks are to be at hand, each having a small hole at the center. By use of a small bolt and wing nut one of these is fastened across the suspended stick at its center $C$ so as to form a plus sign, as in Fig. 1($d$). We have just seen that the cylinder, fastened here, changes the period. With surprise we now find that the plus sign, swinging in its own plane, has the same period as the single vertical stick. It keeps step with the simple pendulum 66.7 cm long, so that the equivalent length is still 66.7 cm. We find, moreover, that we can clamp two meter sticks, or even more than two, across the suspended stick at its center, and the period remains the same. We can even turn the cross sticks at any angle—the same angle, as in Fig. 1($e$), or at different angles—and produce no change in the period $T$. It is striking to find that the cylinder at $C$ markedly changes $T$, whereas the meter sticks crossed at $C$ leave $T$ unchanged. This, of course, is for motion in the plane of the rods. If we clamp the two sticks across and normal to the suspended one and *parallel to the axis*, then their effect is that of the cylinder. They leave the period unchanged if put at $O$, but not when attached at $C$.

Suppose we fasten a thread to each end of a meter stick and suspend the stick from the axis so that it swings alone just as it does when clamped across the vertical stick at its center. This arrangement is shown in Fig. 1($f$), where the dashed lines represent the thread. If the center of the suspended stick is 50 cm below the axis, whether the stick is horizontal or inclined at any angle, as in Fig. 1($g$), the length of the equivalent simple pendulum is 66.7 cm, as before.

The behavior of the horizontal stick alone suggests a general principle. If two pendulums have the same period when swinging independently and separate, the pendulum formed by fastening the two together (in their equilibrium positions, and so that the two axes are in the same line) has also the same period.

The proof is simple. Let $l_e[=I_A/Mh]$ and $l_e'[=I_A'/M'h']$ refer to the two separate pendulums, and let $l_e''[=I_A''/$

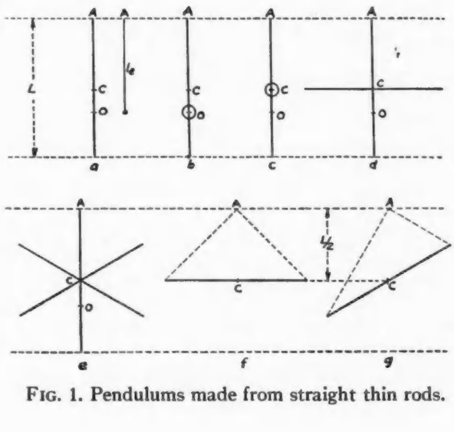


FIG. 1. Pendulums made from straight thin rods.

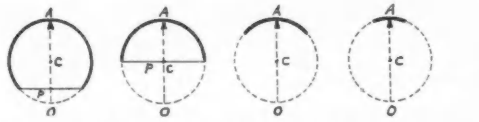


FIG. 2. Different segments of similar rings have the same period; $l_e=2R$ in each case. For purposes of demonstration, light straight sticks $p$ should be fitted between the ends of the longer segments, so that they will retain their circular shape.

$M''h'']$ refer to the combination. Then $l_e=l_e'$, so that $l_e=I_A/Mh=I_A'/M'h'=(I_A+I_A')/(Mh+M'h')$. But $I_A+I_A'=I_A''$, and $h''=(Mh+M'h')/(M+M')=(Mh+M'h')/M''$. Therefore, $l_e=I_A''/M''h''=l_e''$. This equation states that the equivalent length of the combination is the same as that of either component, which was to be proved. To verify the proposition, made obvious as a good approximation by the experiment, that the period of the plus-sign pendulum is the same as that of its vertical component alone, it is sufficient to use the general proposition just established and to note that, for the horizontal stick, $l_e$ is the same as that for the vertical stick, the two lengths being equal. The two masses may be the same or different.

It is one of the important properties of the expression for the coordinate of the center of mass, which may be written $X_G=\Sigma(Mx)/\Sigma M$, that even in the same problem some of the $x$-coordinates may refer to particles and some may be the coordinates of the centers of mass of extended component objects, so that this equation may be used to find the position of the center of mass of (i) a system of particles, (ii) a system of extended bodies, the locations of the individual centroids being known, or (iii) a system made up of particles and extended bodies. This feature of the equation is commonly taken for granted but seldom explicitly stated.

The plus-sign pendulum and the three sticks at angles of 60°, or even at unequal angles, make good problems for the general physics course.

### Circular Rings

One of the problems given by Millikan, Roller and Watson[1] requires the length of the equivalent simple pendulum if a thin wire, bent to form a semicircle of radius $R$, swings in its own vertical plane about an axis at the mid-point of the wire. The answer given is $2R$. It is a little surprising that a semicircular wire should oscillate with the same period as the entire circle of wire of the same radius, since their centroids are at different points and their moments of inertia with respect to the common axis are quite different; but such is the case.

However, this problem leads one to several things more surprising still. In Fig. 2 are shown various arcs of similar rings, each free to oscillate in its own vertical plane about an axis $A$ at its mid-point. They *all* have the same period: the length of the equivalent simple pendulum is $2R$ for each one. In Fig. 3 are shown various circular arcs of the same radius with the two ends joined by a light cord, and each suspended from an axis lying on the circle. Since the centers of oscillation and suspension of any pendulum are interchangeable (without change of period), these arcs have the same period as before, and in each case $l_e=2R$. We arrive at the same conclusion if we use the general principle proved in the first section. For if the first arc in Fig. 2 has the same period as the whole ring, then the piece removed, if it swings about the same axis (suspended by light cords), must have the same period also. Then since the size of the arc removed has no effect on the period of the part remaining, the periods of the parts removed are the same also. All such arcs as are shown in Fig. 3 yield the same value of $l_e$, namely, $2R$.

For lecture demonstrations one can use three or four iron rings, all of the same size, such as once were common as buggy tires. Preserve one entire, and divide the others into arcs of different sizes, one preferably a half circle, and one very

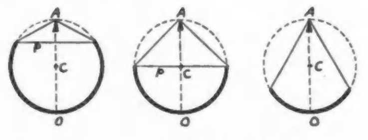


FIG. 3. In each case $l_e=2R$. Light sticks $p$ are used as indicated with Fig. 2.

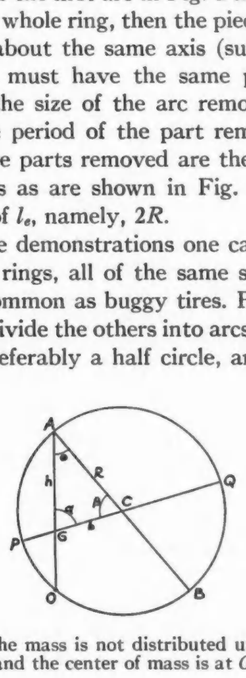


FIG. 4. Here the mass is not distributed uniformly along the arc and the center of mass is at $G$, not $C$.

[1] Millikan, Roller and Watson, *Mechanics, molecular physics, heat and sound* (Ginn, 1937), p. 433, problem 135.

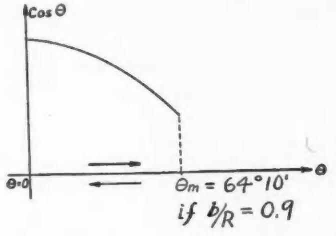


FIG. 5. The value of $\theta$ varies from zero to a maximum of 64° 10′, then back to zero, and $l_e$ varies from a maximum of $2R$ to a minimum of $0.872R$, then back to $2R$.

short. Fishline, which does not stretch much, or fine steel wire, makes a good suspension. A small notch carefully filed as near as may be to the middle of each piece makes it easy to suspend them at their centers. This notch may have to be filed by trial in the shortest piece, for in this one it will have to be of the correct depth. One whole circle and five or six different pieces, swinging in perfect unison, as indicated in Figs. 2 and 3, make an impressive demonstration. They may be on different axes or all on the same axis.

In the pendulums described the mass is uniformly distributed along the arc and the axes are at the mid-points. These conditions are by no means necessary in order that $l_e$ should be equal to $2R$. Allowing any distribution of mass whatever along a circular arc of radius $R$, and using any axis intersecting the arc and perpendicular to its plane, we now determine what is required in order that $l_e$ should be equal to $2R$.

The center of mass will be somewhere inside the circle, as at $G$ in Fig. 4. Here $C$ is the center of the circle and the axis is at $A$. Then the circle will hang in equilibrium with the line $AG$ vertical and making the angle $\theta$ with the diameter $ACB$. To find what $l_e$ is under these circumstances, we have

$$I_C = I_G + Mb^2 = MR^2,$$

where $M$ is the total mass, for this mass, however distributed along the circumference, is all at the distance $R$ from $C$. Also

$$I_A = I_G + Mh^2,$$

where $h$ is the distance from the axis to the centroid. Therefore,

$$I_A = MR^2 - Mb^2 + Mh^2 = M(R^2 + h^2 - b^2).$$

But in the triangle $AGC$, $b^2 = R^2 + h^2 - 2Rh\cos\theta$. Therefore,

$$I_A = 2MRh\cos\theta,$$

and hence

$$l_e = I_A/Mh = 2R\cos\theta.$$

Since $2R\cos\theta$ is equal to the chord $AO$, *the center of oscillation lies on the circumference of the circle*. This is true for any distribution of mass whatever.

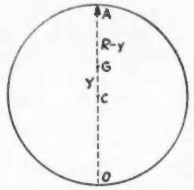


FIG. 6. Pendulum of Fig. 4 for the special case in which the center of mass $G$ is on the diameter $ACO$.

We may state the general rule by saying that, for a given distribution of mass and hence with $G$ definitely located, if the axis is on the circumference the center of oscillation $O$ is on the circumference also. Obviously, if we extend the line $GC$ in Fig. 4 to form the diameter $PQ$, the axis $A$ may be anywhere along one semicircumference from $P$ to $Q$. It is not necessary to follow $A$ through the other half ($POBQ$), since equilibrium cannot occur when $G$ is above $A$. Hence $\theta$ changes from zero (when $A$ is at $P$) to some maximum value, then changes to zero again (when $A$ is at $Q$). With $A$ at $P$ or at $Q$, $\cos\theta$ has its maximum value, namely, unity, and $l_e = 2R$.

We can now answer our original question with complete generality. With the axis at a fixed point of the ring, $l_e = 2R$ if the center of mass $G$ is on the axial diameter $ACB$; $G$ may be above $C$ or below. With $G$ not on the diameter $ACB$, the ring is not symmetrical with respect to $AG$, but the center of oscillation still lies on the circumference.

It is interesting to inquire what maximum value $\theta$ will have as $A$ moves along the arc from $P$ to $Q$. Taking a derivative of $\theta$, or using obvious geometrical relations if we prefer not to differentiate, we find that $\theta$ is a maximum when $\alpha$ is 90°. In that case $\sin\theta = b/R$. If $b/R = 0.9$, for example, the maximum value of $\theta$ is about 64° 10′, then $\cos\theta = 0.436$, and the minimum value of $l_e$, which is $2R\cos\theta$ in general, is $0.872R$. The variation of $\cos\theta$ and $l_e$, from zero

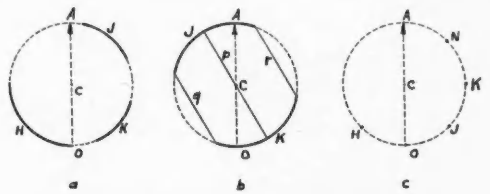


FIG. 7. Pendulums in which the distribution of mass is not symmetrical with respect to the axial diameter, though the center of mass is still on that diameter. Hence the axial diameter is still vertical.

up to 64° 10′ and back to zero for $\theta$, and from $D$, or $2R$, down to $0.436D$, and back up to $D$ for $l_e$, are shown in Fig. 5.

It is a good elementary exercise to find the expression for $l_e$ in the special case for which $G$ lies on the diameter $ACB$, without making use of the general equation $l_e = 2R\cos\theta$. In Fig. 6, the centroid $G$ is above $C$, $GC = y$ and $AG = h = R - y$. Then

$$I_C = MR^2 = I_G + My^2,$$

and

$$I_A = I_G + M(R-y)^2,$$

so that

$$I_A = MR^2 - My^2 + M(R-y)^2$$
$$= 2MR(R-y) = 2MRh.$$

Therefore, $l_e = I_A/Mh = 2R$, as given for the case $\theta = 0$ by the general formula. The argument is the same if $G$ is below $C$.

By tying shorter pieces of tire $H$, $J$, $K$ (Fig. 7) to one complete ring, so that the pieces and the ring are side by side, we can show experimentally that distribution symmetrical with respect to a vertical diameter is not necessary in order that $l_e$ should be equal to $2R$. We could construct the pendulum shown in Fig. 7($b$) with the pieces $J$ and $K$ only, if we could manage to hold them apart with a light stick $p$ and connect the ends with taut cords $q$ and $r$. Thin straight rods parallel to the axis and of any length whatever, fastened to the complete ring as shown at $H$, $J$, $K$ and $N$, Fig. 7($c$), have just the effect of heavy particles. In fact, a brass rod in two equal parts that can be screwed together, 8 or 10 in. long and 1 in. or less in diameter, can be used with a wire ring to demonstrate the unsymmetrical pendulum.

## The Publication Records of Certain American Physicists

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SEVERAL studies have recently appeared on the ages, origins and educational histories of American physicists.[1] No work appears to have been done, however, on their literature productivity. It is the purpose of the writers to present data on the number of scholarly papers in physics published by the members of a select group of physicists and to give the educational backgrounds of these men.

Because of the formidable task involved in a study of the literary productivity of all physicists, it was thought best to restrict such a study to those physicists starred[2] in *American Men of Science*. The productivity of these men, by definition, will be measured by the number of papers abstracted under their names in *Science Abstracts*. The limitations of an attempt to measure objectively the worth to physics of contributions by contemporary physicists are immediately obvious. Mere numbers of books and articles constitute but one factor in the estimation of a physicist's contribution to his science. Future evaluations will be made on bases we cannot entirely foresee. The writers recognize the inadequacy of the standard but feel that the number of papers abstracted under an individual's name will bear a high correlation to the worth of his contribution by whatever standard might be applied.

The present work attempts to supply information on such questions as the following. Do the most noteworthy—that is, starred—physicists obtain their bachelor's degrees from small colleges or large universities? Does any one institution produce an extraordinarily large number of starred men? Is any one university outstanding in graduating Ph.D.'s who publish extensively? How many papers does the average starred physicist publish each year? It is not claimed

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[1] R. Birge, *Am. J. Physics* 9, 24 (1941); O. Blackwood, *Am. J. Physics* 12, 149 (1944); R. M. Bell, *Am. J. Physics* 14, 396 (1946); M. H. Trytten, *Am. J. Physics* 15, 330 (1947).

[2] The preface to the first edition states that a star means that the subject of the biographical sketch is probably among the leading students of science in the United States whose work is supposed to be the most important. For each science, the starred men are selected by ten leading students of that science. The names of the members of the committee are not revealed by the Editor.

that a satisfactory answer has been obtained to each of these questions. One difficulty lies in the small number of individuals involved. However, it is believed that the results to be described are of sufficient interest to warrant presentation, and the reader may draw his own conclusions from the data.

## Procedure

The men selected for the survey were those physicists starred in the seventh edition (1944) of *American Men of Science*. Their names were obtained through a page-by-page search during which a list was compiled of those starred who gave *physics* or *astrophysics* as their field. The names of 250 physicists were found to be starred. During this process, the names of the institutions that granted the starred men their Bachelor's, Master's and Doctor's degrees, together with the year in which the degree was obtained, were listed.

The "Author's Index" of *Science Abstracts, Section A, Physics*, was next consulted for each year from 1898 to 1942, inclusive. For each of the starred physicists, a record was made of the number of articles abstracted[3] for each year of that man's working life. In the case of men possessing an earned Ph.D. degree, the working life was taken from and included the year in which the degree was obtained and extended through 1942. For those older[4] men who received an A.B. or its equivalent but not a Ph.D., the active period was assumed as beginning during the year in which the bachelor's degree was received and extending through 1942. Since the 1944 edition of *American Men of Science* lists only men living at the time of its publication, the 45-year period covers substantially the entire working period of the oldest physicists included. For the younger men, only the early portion of their careers is covered, of course. No attempt has been made to ascertain which period of a man's life has been most productive in numbers of papers published.

Although this procedure fails to consider any publishing done before the Ph.D. had been obtained, it was noted that very few published before the year in which their degree was obtained.[5] On the average, the starred men who were granted doctorates had 0.7 papers per man published before they received their degrees. The survey was not extended beyond 1942 because many physicists had gone into war projects about that time, nor before 1898 which was the inaugural year of *Science Abstracts*. No effort was made to determine when a man retires from publishing, and in all cases the active period includes 1942. Full credit for a paper was given to a man even though he was a co-author.

TABLE I. Institutions granting bachelor's degrees to four or more physicists who were later starred.

| Institution | Number of starred men granted bachelor's degrees | Total number* of bachelor's degrees granted | Ratio of number starred to number graduated |
|---|---|---|---|
| Chicago | 10 | 41 | 0.24 |
| Massachusetts Institute | 10 | 74 | .14 |
| Yale | 10 | 32 | .31 |
| Minnesota | 9 | 26 | .35 |
| Johns Hopkins | 8 | 31 | .26 |
| Cornell | 7 | 54 | .13 |
| California Institute | 6 | 34 | .18 |
| Harvard | 6 | 46 | .13 |
| Princeton | 6 | 27 | .22 |
| Wisconsin | 6 | 50 | .12 |
| California | 5 | 37 | .14 |
| Columbia | 5 | 30 | .17 |
| Toronto | 5 | 35 | .14 |
| Case | 4 | 23 | .17 |
| Michigan | 4 | 59 | .07 |
| Oberlin | 4 | 41 | .10 |

* Taken from R. M. Bell, reference 1, Table II. Only those graduates listed in *American Men of Science* (1944) were included.

## Results

Table I lists the institutions from which four or more starred physicists received the bachelor's degree. Ten colleges, not listed, graduated three starred men each; 10, two men each; and 56, one man each. Others of the group were educated outside the United States or received no bachelor's degree. As might be expected, the larger colleges (in enrolment) from which also[6] come large numbers of physicists, show the larger numbers of starred men per institution. Considering all the colleges granting a bachelor's

[3] Not necessarily submitted or published during that year. Books or popular articles are not included, of course.

[4] Only men possessing the doctorate have been elected to a starred place in recent years, it was found.

[5] A singular exception is that of I. S. Bowen, who published 14 papers in the six years intervening between the granting of his A.B. and Ph.D. degrees.

[6] O. Blackwood, reference 1.

TABLE II. Institutions granting the doctorate to six or more physicists who were later starred.

| Institution | Number of Ph.D.'s who became starred | Total Ph.D.'s* granted in physics | Ratio of number starred to total |
|---|---|---|---|
| Johns Hopkins | 32 | 130 | 0.25 |
| Princeton | 20 | 74 | .27 |
| Chicago | 18 | 179 | .10 |
| Cornell | 16 | 149 | .11 |
| Harvard | 15 | 91 | .17 |
| California Institute | 12 | 111 | .11 |
| Columbia | 12 | 88 | .17 |
| Minnesota | 10 | 34 | .29 |
| Yale | 9 | 80 | .11 |
| Michigan | 7 | 97 | .07 |
| Clark | 6 | 18 | .33 |
| Wisconsin | 6 | 83 | .07 |
| California | 6 | 116 | .05 |

* Taken from R. M. Bell, reference 1, Table III. Only those Ph.D. graduates listed in *American Men of Science* (1944) were included.

degree to men who were later starred, it was found that nearly half the starred men received their undergraduate training at institutions where Ph.D. work was available. The fourth column of Table I gives the ratio of the number of physicists graduated with a bachelor's degree who were later starred to the total number of physics degrees granted. By this criterion of excellence, Minnesota and Yale are outstanding.

The institutions granting the doctorate to six or more starred men are listed in Table II. Several German Universities might be added—Göttingen with six men starred, Munich with five, and Leipzig and Berlin with four each. Twenty-three foreign universities granted doctorates to 46 men who are starred. Johns Hopkins leads with a total of 32 Ph.D. graduates starred. However, in the percentage of those students graduated with the doctorate who became starred, Clark stands highest with 33 percent, followed by Minnesota, Princeton, and Hopkins, closely grouped. One is tempted to explain the order in column four of Table II as due in part to the fact that older men are more likely to be starred than younger. Thus, Clark is first and the two California institutions, which have a relatively high proportion of young Ph.D. graduates,[7] are low in the ratio of starred to total. It was found that 19 of the 250 starred men never received an earned Ph.D. degree. Of the 53 graduate schools in the United States offering the Ph.D. in physics, only 24 have trained men who were later starred.

[7] R. M. Bell, reference 1.

TABLE III. Papers published per starred graduate per year for those schools having six or more Ph.D. graduates starred.

| Institution | Number of Ph.D. graduates starred | Papers per man year |
|---|---|---|
| California | 6 | 1.7 |
| California Institute | 12 | 1.6 |
| Wisconsin | 6 | 1.2 |
| Harvard | 15 | 1.0 |
| Yale | 9 | 1.0 |
| Princeton | 20 | 0.96 |
| Minnesota | 10 | .92 |
| *No doctorate* | 19 | .92 |
| Chicago | 18 | .88 |
| Columbia | 12 | .87 |
| Michigan | 7 | .84 |
| Johns Hopkins | 32 | .81 |
| Göttingen | 6 | .80 |
| Cornell | 16 | .75 |
| Clark | 6 | .21 |

The scholarly production of the starred men is indicated by the summary by graduate schools found in Table III. This table was obtained as follows. For each man, a compilation was made of the number of papers abstracted during each calendar year of that individual's working period. Individual records were then collected under their respective alma maters, and the averages which appear in column three were drawn up.

The average output of articles by a starred man was found to be less than one (0.93) per year, considering all 250. From Table III, it may be seen that the starred graduates of California and California Institute nearly doubled this average output. It is interesting to point out that although the two California schools had a low percentage of their graduates starred, those who were starred stand at the top in the number of papers published each year. It will also be noted from Table III that *the average production of the 19 men who do not hold a doctorate equals the average of all the 250.*

During the collection of the data contained in the three tables, some additional facts came to light. *Eight men had been starred who had never had a paper abstracted. Three others were starred who had written only one paper.*

The most prolific worker in terms of total papers published in the period studied was found to be R. W. Wood, with 182. He is followed in order by Coblentz, Ives, O. W. Richardson,

Bridgman, Swann and Millikan, all of whom produced more than 100 papers since receiving the Ph.D. Those publishing more than 80 but fewer than 100 were Einstein, Tolman, Meggers, Hulburt and Breit. Those publishing between 60 and 80 were Fermi, A. H. Compton, K. T. Compton, R. S. Mulliken, Herzfeld, Landé, Loeb, H. A. Wilson, Landenburg, Mohler and A. S. King. The most prolific workers during single years were R. W. Wood who had 13, 12 and 11 papers in three years and H. E. Ives who had 13 and 12 in successive years. Bethe had 10 and 9 in two years, Foote 11 and 9, Bridgman and R. S. Mulliken 11 in one year, and Landé 10.

Some interest may be derived if, instead of totals, we consider the average time-rate of production. Here again, Wood stands at the top accompanied by Bethe, each with 4.0 papers per year on the average. Next in order are Breit (3.7), Fermi (3.5), Ives (3.5), Coblentz (3.5), Bridgman (3.3), Swann (3.3), R. S. Mulliken (3.0) and Bowen (3.0). Thus, the list of men performing at a high rate is almost identical with the list of men high in total papers. This need not necessarily be so, and indeed we find Bethe and Bowen, who are comparatively young, producing at the rate of four and three papers per year, respectively. Looking at the list of men high in papers per year, one sees mostly names of older men, but whether or not this means that papers were "easier" to write in the past is a question we must leave unanswered.

Among the starred physicists, there occur three pairs of brothers, the Comptons, Stewarts and Zelenys. Three women are listed. Every starred physicist holds either a Bachelor's or a Ph.D. degree or both.

In conclusion, it might be repeated that the information supplied is only for starred physicists listed in *American Men of Science*. It covers only those living in 1944. It should be remembered that some institutions are represented by but few men. Thus, certain factors may have a large effect on the records of certain schools. For example, should a physicist elect to go into industry or take over administrative duties, his publishing might be curtailed. Also, it is possible that the branch of physics selected for the doctoral dissertation will influence the quantity of later research. And there is a case of one starred man who received his Ph.D. in physics turning to physiology, in which field he published extensively. Only a very comprehensive study could take into account the many factors involved and answer the questions possibly suggested here.

## Foreign Physicists Available

Many physicists and engineers now living in D.P. (Displaced Persons) Camps in Germany are eager to obtain positions on American campuses. These men and women represent various nationalities and religions. Individual reports have been prepared that describe their training, experience, age, size of family, and so forth, and include comments by an American interviewer on their personality and command of English.

A one-year contract is all that is necessary to bring a D.P. professor to an American campus. Address inquiries to Miss Mary Burke, War Relief Service, 350 Fifth Avenue, New York 1, N. Y.

# Modern Terminology for Physics

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THE terminology of physics has developed over a period of several centuries, and as a consequence it is not always ideally suited to modern needs. Some of the 19th century concepts of magnetism, for instance, are antithetical to modern theory. The concepts of photometry, to cite another example, were founded by technicians who had little interest in the broad aspects of light. The result is a lack of generality in the older treatments of photometry and radiometry.

Names, also, can be improved in many cases. Some of the older names—such as *force*, *work* and *charge*—are ambiguous and are themselves a cause of confusion and an incitement to inexact thinking. Other names, because of their provincial character, are becoming more and more troublesome, as improved means of world communication emphasize internationality.

A modern treatment of radiometry and photometry was suggested in previous papers.[1] In the present study, the same principles are applied to classical physics. Concepts, names, dimensions and units are considered. In many cases, the proposed names are those in common use; but when the ordinary names are cumbersome, ambiguous or noninternational, words have been selected from classical Greek. The dimensions are in accordance with the results of another study.[2] The use of mks units throughout has a great advantage in simplifying and unifying the presentation of physical principles.[3] There is, of course, nothing *official* about the present paper. It merely offers some suggestions based on the personal opinions of the authors.

## Concepts

Table I lists the principal concepts of classical physics. Such ideas as *distance*, *area*, *angle* and *time* are fundamental and need no comment. Table I also includes such mechanical concepts as *mass*, *force*, *energy* and *power*. Notable for their absence are *weight* and *work*. If *weight* were not taught as a scientific concept, the frequent confusion of mass and weight would be eliminated.[4] The ordinary balance is used as a mass comparator, not as an instrument for determining weight. The standard kilogram is a standard of mass, not of weight. Weight is merely a special example of a force. It should never be treated as a distinct concept. These statements are absurdly elementary and obvious; yet even in scientific textbooks, the authors frequently use "mass" and "weight" interchangeably, to the confusion of the student. The remedy is to eliminate the word "weight" in technical discussions.

*Work* is another concept that is omitted from Table I. The use of such an anthropomorphic name for a scientific concept is highly reprehensible.[5] The concept is merely a special case of energy, and it can always be called by that name.

Mechanics includes a large number of concepts, some of which are used infrequently. How many of these concepts should be given distinct names and symbols is a matter of opinion.[5] Evidently one should steer between the one extreme of having a name for every possible combination of quantities, and the other extreme of having only three or four concepts, with everything else considered as combinations of these few fundamental

[1] P. Moon and D. E. Spencer, "Internationality in the names of scientific concepts: a method of naming concepts," *Am. J. Physics* **14**, 285 (1946); "A proposed international photometric system," *Am. J. Physics* **14**, 431 (1946); "Comparison of photometric systems," *Am. J. Physics* **15**, 84 (1947); "A study of photometric nomenclature," *J. Opt. Soc. Am.* **36**, 666 (1946); "Photometric nomenclature for the post-war world," *Illum. Eng.* **42**, 611 (1947); "Brightness and helios," *Illum. Eng.* **39**, 507 (1944); "Brillo y helios," *Revista Electrotecnica* **32**, 353 (1946).

[2] P. Moon and D. E. Spencer, "A modern approach to dimensions," to be published.

[3] P. Moon and D. E. Spencer, "Utilizing the mks system," *Am. J. Physics* **17**, 25 (1948).

[4] L. A. Hawkins and S. A. Moss, "Alice and the sluggers," *Am. J. Physics* **13**, 409 (1945).

[5] P. Moon, "The names of physical concepts," *Am. J. Physics* **10**, 134 (1942); "A system of photometric concepts," *J. Opt. Soc. Am.* **32**, 348 (1942).

concepts. *Impulse* and *action* are often included as concepts of mechanics, but it is questionable whether the additional complexity that they introduce is justified. These and some others have been omitted in Table I.

In thermal theory, the antiquated ideas associated with the *calorie* are still in use. The concept called "quantity of heat" was necessary in the early study of heat, but both the name and the unit (calorie) are out of place in a modern treatment of physics.[6] In Table I, the concept is called *thermal energy* and is expressed in *joules*, like any other form of energy.

The concepts of electricity are well established and need no comment. The older treatment of magnetism in terms of magnetic poles is replaced in most modern textbooks by a treatment based on current. Thus the concept of *magnetic pole* is unnecessary.[7] Magnetic polarization and susceptibility can also be eliminated with profit. Even in non-isotropic media, it would seem better to use the permeability tensor than to employ magnetic susceptibility and polarization.

The concepts of radiometry and photometry are treated elsewhere.[1] In these subjects, as in the other branches of physics, the attempt is made to use a minimum number of concepts. A great pedagogic advantage results from concentrating on a few fundamental ideas, on a single preferred method of calculation, on one system of units. In his advanced work in science and engineering, the student can bring in all the additional ideas he needs. But lucky is he who has acquired his knowledge of basic principles without the confusing multiplicity and ambiguity usually associated with elementary physics!

## Names

As stated previously,[5] the requirements for a good name of a scientific concept are (i) nonambiguity, (ii) internationality, (iii) simplicity, (iv) euphony. The first names (distance, area, volume, time) in Table I do not satisfy the requirement of internationality, though they are otherwise satisfactory. The concepts, however, are so closely related to everyday life that it seems inadvisable to replace their established names by international ones. This procedure has been used also for the names of other kinematic concepts.

In dynamics, the concepts are not so closely associated with everyday living, and the use of the common words violates both the requirement of nonambiguity and that of internationality. It is true that *mass* and *energy* are international words that satisfy all four requirements. But *force* and *power* have widely different forms in different languages[8] and are therefore noninternational. We propose the Greek words[5] *kratos* and *dynamos*[9] to replace "force" and "power." The new words are short and easy to remember. As shown previously,[1] they could be accepted without translation by the languages of Europe and the Americas. The name "moment of inertia" is cumbersome and noninternational. It could well be replaced by the short word *argos*. The derivation of these and other proposed words is given in Table II.

In the subject of heat, *temperature* and *entropy* are international and should not be changed. We suggest the name *thermitivity*, however, in place of specific heat. The new word ends in -ITY to show that the concept represents a *passive property of a material*.[1] *Conductivity* and *diffusivity* are also properties of materials and therefore have the correct ending.[10] In *emissivity*, however, the ending is incorrect and the same name is employed for several other scientific concepts. As used here, it is the proportionality constant that relates thermal power, dissipated from a unit area of surface, with the temperature drop at the surface. Evidently this concept is not a characteristic of the material alone but depends also on the condition of the surface and the velocity and nature of the cooling medium. The word-ending should be -ANCE (as in resistance, inductance,

[6] E. F. Mueller and F. D. Rossini, "Calory and the joule in thermodynamics and thermochemistry," *Am. J. Physics* **12**, 1 (1944).

[7] F. W. Warburton, "The magnetic pole, a useless concept," *Am. J. Physics* **2**, 1 (1934); D. L. Webster, "Facing reality in the teaching of magnetism," *Am. J. Physics* **2**, 7 (1934).

[8] English, *force;* French, *force;* German, *Kraft;* Russian, *sīla*. English, *power;* French, *puissance;* German, *Leistung;* Russian, *moshchnost*.

[9] The vowels are pronounced as in most European languages: *a* as in *father*, *e* as in *met*, *i* as in *machine*, *o* as in *for*, *u* like *oo* in *soon*. The ending -AGE is pronounced *-azh* as in French. Words ending in -OS are accented on the penultimate (except *hel' i os*); words ending in -ENT, -AGE or -UM are stressed on the final syllable.

[10] American Standards Assoc., "Letter symbols for heat and thermodynamics, including heat flow," Z10.4 (1943).

TABLE I. Proposed terminology.

| Symbol | Common name | Proposed name | Defining equation | Dimensions $[ML_rL_tT\Theta QF]$ | Dimensions $[PL_rL_tT\Theta IF]$ | Mks unit |
|---|---|---|---|---|---|---|
| | | | *Kinematics* | | | |
| $s$ | distance | distance | | $[L_r]$ | $[L_r]$ | m |
| $A$ | area | area | $A=s^2$ | $[L_t^2]$ | $[L_t^2]$ | $m^2$ |
| $V$ | volume | volume | $V=s^3$ | $[L_rL_t^2]$ | $[L_rL_t^2]$ | $m^3$ |
| $t$ | time | time | | $[T]$ | $[T]$ | sec |
| $v$ | velocity | velocity | $v=ds/dt$ | $[L_rT^{-1}]$ | $[L_rT^{-1}]$ | m $sec^{-1}$ |
| $a$ | acceleration | acceleration | $a=dv/dt$ | $[L_rT^{-2}]$ | $[L_rT^{-2}]$ | m $sec^{-2}$ |
| $\theta$ | angle | angle | $\theta=s/r$ | $[L_tL_r^{-1}]$ | $[L_tL_r^{-1}]$ | radian |
| $\Omega$ | solid angle | solid angle | $\Omega=dA/r^2$ | $[L_t^2L_r^{-2}]$ | $[L_t^2L_r^{-2}]$ | steradian |
| $\omega$ | angular velocity | angular velocity | $\omega=d\theta/dt$ | $[L_tL_r^{-1}T^{-1}]$ | $[L_tL_r^{-1}T^{-1}]$ | rad $sec^{-1}$ |
| $\alpha$ | angular acceleration | angular acceleration | $\alpha=d\omega/dt$ | $[L_tL_r^{-1}T^{-2}]$ | $[L_tL_r^{-1}T^{-2}]$ | rad $sec^{-2}$ |
| $\lambda$ | wavelength | wavelength | | $[\Lambda]$ | $[\Lambda]$ | micron |
| $t$ | period | period | | $[T]$ | $[T]$ | sec |
| $f$ | frequency | frequency | $f=1/\text{period}$ | $[T^{-1}]$ | $[T^{-1}]$ | hertz |
| | | | *Dynamics* | | | |
| $m$ | mass | mass | | $[M]$ | $[PL_r^{-2}T^3]$ | kg |
| $F$ | force | kratos | $F=ma$ | $[ML_rT^{-2}]$ | $[PL_r^{-1}T]$ | newton |
| $\mathcal{T}$ | torque | torque | $\mathcal{T}=\mathbf{F}\times\mathbf{s}$ | $[ML_rL_tT^{-2}]$ | $[PL_r^{-1}L_tT]$ | newton m |
| $\mathcal{I}$ | moment of inertia | argos | $\mathcal{I}=\int r^2dm$ | $[ML_r^2]$ | $[PT^3]$ | kg $m^2$ |
| $U$ | energy | energy | | $[ML_r^2T^{-2}]$ | $[PT]$ | joule [=watt sec] |
| $P$ | power | dynamos | $P=dU/dt$ | $[ML_r^2T^{-3}]$ | $[P]$ | watt |
| $T$ | normal stress | normal kratosage | $T=F/A$ | $[ML_rL_t^{-2}T^{-2}]$ | $[PL_r^{-1}L_t^{-2}T]$ | newton $m^{-2}$ |
| $T$ | shear stress | shear kratosage | $T=F/A$ | $[ML_t^{-1}T^{-2}]$ | $[PL_r^{-2}L_t^{-1}T]$ | newton $m^{-2}$ |
| $S$ | normal strain | normal kampulos | | 0 | 0 | numeric |
| $S$ | shear strain | shear kampulos | $S=\Delta s/s$ | $[L_rL_t^{-1}]$ | $[L_rL_t^{-1}]$ | numeric |
| $D$ | density | density | $D=m/V$ | $[ML_r^{-1}L_t^{-2}]$ | $[PL_r^{-3}L_t^{-2}T^3]$ | kg $m^{-3}$ |
| | | | *Heat* | | | |
| $T$ | temperature | temperature | | $[\Theta]$ | $[\Theta]$ | deg |
| $U$ | thermal energy | thermal energy | | $[ML_r^2T^{-2}]$ | $[PT]$ | joule [=watt sec] |
| $P$ | thermal power | thermal dynamos | $P=dU/dt$ | $[ML_r^2T^{-3}]$ | $[P]$ | watt |
| $c$ | specific heat | thermitivity | $c=(1/m)dU/dT$ | $[L_r^2T^{-2}\Theta^{-1}]$ | $[L_r^2T^{-2}\Theta^{-1}]$ | joule $kg^{-1}$ $deg^{-1}$ |
| $k$ | thermal conductivity | thermal conductivity | $k=P/(AdT/ds)$ | $[ML_r^3L_t^{-2}T^{-3}\Theta^{-1}]$ | $[PL_rL_t^{-2}\Theta^{-1}]$ | watt $m^{-1}$ $deg^{-1}$ |
| $h^2$ | diffusivity | diffusivity | $h^2=k/cD$ | $[L_r^2T^{-1}]$ | $[L_r^2T^{-1}]$ | $m^2$ $sec^{-1}$ |
| ə | emissivity | thalpance | ə $=P/AT$ | $[ML_r^2L_t^{-2}T^{-3}\Theta^{-1}]$ | $[PL_t^{-2}\Theta^{-1}]$ | watt $m^{-2}$ $deg^{-1}$ |
| $\mathcal{S}$ | entropy | entropy | $\mathcal{S}=\int dQ/T$ | $[ML_r^2T^{-2}\Theta^{-1}]$ | $[PT\Theta^{-1}]$ | joule $deg^{-1}$ |
| | | | *Electrostatics* | | | |
| $Q$ | charge | electros | | $[Q]$ | $[IT]$ | coulomb |
| ↄ | charge per unit length | electrosent | ↄ $=Q/s$ | $[QL_r^{-1}]$ | $[ITL_r^{-1}]$ | coul $m^{-1}$ |
| ч | charge per unit area | electrosage | ч $=Q/A$ | $[QL_t^{-2}]$ | $[ITL_t^{-2}]$ | coul $m^{-2}$ |
| ʎ | charge per unit volume | electrosum | ʎ $=Q/V$ | $[QL_r^{-1}L_t^{-2}]$ | $[ITL_r^{-1}L_t^{-2}]$ | coul $m^{-3}$ |
| $E$ | electric field strength | electric zenos | $E=dF/dQ$ | $[ML_rT^{-2}Q^{-1}]$ | $[PL_r^{-1}I^{-1}]$ | watt $m^{-1}$ |
| $D$ | electric flux density | electric phantosage | $\int\mathbf{D}\cdot d\mathbf{a}=Q$ | $[QL_t^{-2}]$ | $[ITL_t^{-2}]$ | coul $m^{-2}$ |
| $\epsilon$ | permittivity | permittivity | $\epsilon=D/E$ | $[M^{-1}L_r^{-1}L_t^{-2}T^2Q^2]$ | $[P^{-1}I^2L_rL_t^{-2}T]$ | farad $m^{-1}$ |
| $\varphi$ | potential | potential | $\varphi=\int\mathbf{E}\cdot d\mathbf{s}$ | $[ML_r^{-2}T^{-2}Q^{-1}]$ | $[PI^{-1}]$ | volt |
| $V$ | potential difference | potential difference | $V=\varphi_A-\varphi_B$ | $[ML_r^2T^{-2}Q^{-1}]$ | $[PI^{-1}]$ | volt |
| $C$ | capacitance | capacitance | $C=Q/V$ | $[M^{-1}L_r^{-2}T^2Q^2]$ | $[I^2TP^{-1}]$ | farad |

reluctance) to indicate a passive property of an entity.[9] The suggested word is *thalpance*.

Several new words are proposed for the subject of electrostatics. In place of the ambiguous word "charge," the word *electros* is suggested.[5] The endings -ENT, -AGE and -UM then allow the exact designation of the three charge densities.[11,12] In place of "field strength," or "field intensity," we suggest the simple word *zenos*. This word can be used for any kind of field, the proper adjective being employed in each case. Also useful in any kind of field are the words *phantos* for flux and *phantosage* for flux density. *Potential*, *capacitance* and *permitivity* are already international and need not be altered.

In electrodynamics, the words "current" and "current density" could well be replaced by *hermos* and *hermosage*. The other words in this section of the table satisfy all requirements. For

[11] "Pri la defino e nomizo di ciencala koncepti," *Progreso* **23**, 13 (1947).

[12] P. Moon and D. E. Spencer, *An unofficial guide to photometric nomenclature*, to be published.

TABLE I.—*Continued.*

| Symbol | Common name | Proposed name | Defining equation | Dimensions $[ML_rL_tT\Theta QF]$ | Dimensions $[PL_rL_tT\Theta IF]$ | Mks unit |
|---|---|---|---|---|---|---|
| | | | *Electrodynamics* | | | |
| $I$ | current | hermos | $I=dQ/dt$ | $[QT^{-1}]$ | $[I]$ | ampere |
| $u$ | current density | hermosage | $u=dI/dA$ | $[QL_t^{-2}T^{-1}]$ | $[IL_t^{-2}]$ | amp $m^{-2}$ |
| $\rho$ | resistivity | resistivity | $\rho=RA/l$ | $[ML_rL_t^2T^{-1}Q^{-2}]$ | $[PL_r^{-1}L_t^2I^{-2}]$ | ohm m |
| $\sigma$ | conductivity | conductivity | $\sigma=1/\rho$ | $[M^{-1}L_r^{-1}L_t^{-2}TQ^2]$ | $[P^{-1}L_rL_t^{-2}I^2]$ | mho $m^{-1}$ |
| $R$ | resistance | resistance | $R=V/I$ | $[ML_r^2T^{-1}Q^{-2}]$ | $[PI^{-2}]$ | ohm |
| $G$ | conductance | conductance | $G=1/R$ | $[M^{-1}L_r^{-2}TQ^2]$ | $[I^2P^{-1}]$ | mho |
| | | | *Magnetism* | | | |
| $H$ | magnetic field strength | magnetic zenos | $\oint \mathbf{H}\cdot d\mathbf{s}=NI$ | $[QL_t^{-1}T^{-1}]$ | $[L_t^{-1}I]$ | amp turn $m^{-1}$ |
| $B$ | magnetic flux density | magnetic phantosage | $d\mathbf{F}=Id\mathbf{s}\times\mathbf{B}$ | $[ML_rL_t^{-1}T^{-1}Q^{-1}]$ | $[PTL_r^{-1}L_t^{-1}I^{-1}]$ | weber $m^{-2}$ |
| $\Phi$ | magnetic flux | magnetic phantos | $\Phi=\int \mathbf{B}\cdot d\mathbf{a}$ | $[ML_rL_tT^{-1}Q^{-1}]$ | $[PTL_r^{-1}L_tI^{-1}]$ | weber |
| $\mu$ | permeability | permeability | $\mu=B/H$ | $[ML_rQ^{-2}]$ | $[PTL_r^{-1}I^{-2}]$ | henry $m^{-1}$ |
| $\mathfrak{R}$ | reluctance | reluctance | $\mathfrak{R}=\mathfrak{F}/\Phi$ | $[M^{-1}L_r^{-1}L_t^{-1}Q^2]$ | $[P^{-1}L_rL_t^{-1}I^2T^{-1}]$ | amp turn $weber^{-1}$ |
| $L$ | inductance | inductance | $L=N\Phi/I$ | $[ML_rL_tQ^{-2}]$ | $[PTL_r^{-1}L_tI^{-2}]$ | henry |
| | | | *Radiometry (active concepts)* | | | |
| $F_r$ | radiant power | radiant pharos | | $[ML_r^2T^{-3}]$ | $[P]$ | watt |
| $D_r$ | radiant flux density | radiant pharosage | $D_r=F_r/A$ | $[ML_r^2L_t^{-2}T^{-3}]$ | $[PL_t^{-2}]$ | watt $m^{-2}$ |
| $Q_r$ | radiant energy | radiant phos | $Q_r=F_rT$ | $[ML_rT^{-2}]$ | $[PT]$ | joule [=watt sec] |
| $U_r$ | radiant energy density | radiant phosage | $U_r=Q_r/A$ | $[ML_rL_t^{-2}T^{-2}]$ | $[PL_t^{-2}T]$ | joule $m^{-2}$ |
| $H_r$ | radiant "brightness" | radiant helios | $H_r=\pi \lim_{\Omega\to 0} D_rm/\Omega$ | $[ML_r^4L_t^{-4}T^{-3}]$ | $[PL_r^2L_t^{-4}]$ | herschel |
| $G_r$ | radiant "brightness" gradient | radiant heliosent | $G_r=\lim_{\Delta l\to 0} \Delta H_r/\Delta l$ | $[ML_r^3L_t^{-4}T^{-3}]$ | $[PL_rL_t^{-4}]$ | herschel $m^{-1}$ |
| $J(\lambda)$ | spectral power density | phengosage | $J(\lambda)=\lim_{\Delta\lambda\to 0} \Delta D_r/\Delta\lambda$ | $[ML_r^2L_t^{-2}\Lambda^{-1}T^{-3}]$ | $[PL_t^{-2}\Lambda^{-1}]$ | watt $m^{-2}$ $\mu^{-1}$ |
| | | | *Photometry (active concepts)* | | | |
| $F_l$ | luminous flux | luminous pharos | | $[F]$ | $[F]$ | lumen |
| $D_l$ | luminous flux density | luminous pharosage | $D_l=F_l/A$ | $[FL_t^{-2}]$ | $[FL_t^{-2}]$ | lumen $m^{-2}$ |
| $Q_l$ | quantity of light | luminous phos | $Q_l=F_lt$ | $[FT]$ | $[FT]$ | lumen sec |
| $U_l$ | exposure | luminous phosage | $U_l=Q_l/A$ | $[FTL_t^{-2}]$ | $[FTL_t^{-2}]$ | lumen sec $m^{-2}$ |
| $H_l$ | brightness | luminous helios | $H_l=\pi \lim_{\Omega\to 0} D_lm/\Omega$ | $[FL_t^{-4}L_r^2]$ | $[FL_t^{-4}L_r^2]$ | blondel |
| $G_l$ | brightness gradient | luminous heliosent | $G_l=\lim_{\Delta l\to 0} \Delta H_l/\Delta l$ | $[FL_t^{-4}L_r]$ | $[FL_t^{-4}L_r]$ | blondel $m^{-1}$ |

the magnetic field, *zenos* and *phantos* may be employed. All other terms seem to be satisfactory.

Another word should be mentioned, though usually it does not represent a quantitative concept. That word is "field." It is ambiguous, since it means not only a portion of space having peculiar properties ("electric field," "magnetic field," "light field") but also a sphere of activity, such as "the field of electrical engineering." The English dictionary lists 12 other meanings. The word for "grass plot" is carefully translated into all languages and used as the word to represent this highly specialized scientific concept which has nothing in common with grass plots. In French, it is called *champ;* in Spanish, *campo;* in German, *Feld;* in Russian, *pole;* in Esperanto, *kampo*. Ambiguity and noninternationality would be eliminated by employing a Greek word such as *stereon* (from στερεός, three-dimensional).

Experience has shown that many scientific words taken from ancient Greek are accepted by the entire scientific world, regardless of language. Small changes in orthography are of course desirable to make the word fit into the various ethnic languages. But these changes are so small that the reader knows exactly what is meant

TABLE II. Origins of proposed words.

| Word | Original Greek | Greek meaning |
|---|---|---|
| kratos | κράτος | strength |
| kampulos | καμπύλος | bent |
| argos | ἀργός | idle, lazy |
| dynamos | δύναμις | power |
| thermitivity | θερμός +ITY | hot |
| thalpance | θάλπος +ANCE | warmth |
| electros | ἤλεκτρος | amber |
| stereon | στερεός | solid, three-dimensional |
| zenos | Ζηνός | (of) Zeus, father of the gods |
| phantos | φαντᾰσμα | phantom |
| hermos | Ἑρμῆς | Hermes, messenger of the gods |

TABLE III. Proposed international words.

| Common name, English | English | International | French | Italian | Spanish | Portuguese | German | Russian |
|---|---|---|---|---|---|---|---|---|
| | Proposed names | | | | | | | |
| force | kratos | kratoso | cratosse | cratoso | cratoso | cratoso | Kratos | kratos |
| stress, pressure | kratosage | kratosago | cratosage | cratosaggio | cratosago | cratosago | Kratosag | kratosazh |
| strain | kampulos | kampuloso | campulosse | campuloso | campuloso | campuloso | Kampulos | kampulos |
| moment of inertia | argos | argoso | argosse | argoso | argoso | argoso | Argos | argos |
| power | dynamos | dinamoso | dynamosse | dinamoso | dinomoso | dinomoso | Dynamos | dynamos |
| specific heat | thermitivity | termitivito | thermitivité | termitività | termitividad | termitividade | Thermitivität | termītīvitī |
| emissivity | thalpance | talpanco | thalpance | talpanza | talpancia | talpância | Thalpanz | talpantz |
| electric charge | electros | elektroso | electrosse | electroso | electroso | electroso | Elektros | ėlektros |
| charge per unit length | electrosent | elektrosento | electrosent | electrosento | electrosento | electrosento | Elektrosent | ėlektrosent |
| charge per unit area | electrosage | elektrosago | electrosage | electrosaggio | electrosago | electrosago | Elektrosag | ėlektrosazh |
| charge per unit volume | electrosum | elektrosomo | electrosum | electrosumo | electrosumo | electrosumo | Elektrosum | ėlektrosum |
| field | stereon | stereono | stereon | stereono | estereono | estereono | Stereon | stereon |
| field strength | zenos | zenoso | zenosse | zenoso | zenoso | zenoso | Zenos | zenos |
| flux | phantos | fantoso | phantosse | fantoso | fantoso | fantoso | Phantos | fantos |
| flux density | phantosage | fantosago | phantosage | fantosaggio | fantosago | fantosago | Phantosag | fantosazh |
| current | hermos | hermoso | hermosse | ermoso | hermoso | hermoso | Hermos | ėrmos |
| current density | hermosage | hermosago | hermosage | ermosaggio | hermosago | hermosago | Hermosag | ėrmosazh |
| radiant power* | pharos(radiant) | faroso | pharosse | faroso | faroso | faroso | Pharos | faros |
| radiant flux density* | pharosage | farosago | pharosage | farosaggio | farosago | farosago | Pharosag | farosazh |
| radiant energy* | phos | foso | phosse | foso | foso | foso | Phos | fos |
| radiant energy density* | phosage | fosago | phosage | fosaggio | fosago | fosago | Phosag | fosazh |
| radiant brightness* | helios | helioso | heliosse | elioso | helioso | helioso | Helios | ėlīos |
| radiant brightness gradient* | heliosent | heliosento | heliosent | eliosento | heliosento | heliosento | Heliosent | ėlīosent |
| spectral power density | phengosage | fengosago | phengosage | fengosaggio | fengosago | fengosago | Phengosag | fengosazh |

* The same international words apply also to photometric concepts; see reference 1.

when he encounters the word in any part of the world.

Table III shows how the proposed words look in eight different languages. In the second column are listed the words as written in English, these words being taken from Table I. The third column shows how they appear in international auxiliary languages (Esperanto, Ido, IALA K). The remaining columns are for French, Italian, Spanish, Portuguese, German and Russian. In deference to the limitations of American printing establishments, we have transliterated the Russian, using the Library of Congress method.

## Dimensions and Units

In previous papers,[1] the advantages have been pointed out for a system of dimensions that employs the two lengths $[L_r]$ and $[L_t]$. Dimensions of the recommended concepts are given in this system in column five of Table I. Another system having the same advantages is tabulated in column six. The two are exactly similar except that in column six, power is taken as a fundamental dimension [P], while [M] is omitted. The latter system is advantageous in radiometry, but either may be employed at the discretion of the scientist.

The final column of Table I gives the rationalized mks units. The mks system was introduced primarily for electromagnetism, where it provides a wonderful simplification over the previous sets of units. The mks system, however, is equally applicable to all branches of physics,[13] where it unifies and simplifies the entire subject. The use of a single system of units is of great pedagogic advantage in physics and engineering.

## Summary

The paper presents a proposed terminology for physics. Important concepts are included and redundant ones are omitted. A number of international words are suggested to replace unsatisfactory concept names. The whole set of concepts is unified by means of the mks system of units. We feel that the use of the proposed terminology would simplify the teaching of both physics and engineering subjects and would promote more exact thought and a better grasp of fundamentals.

[13] A. E. Kennelly, "I.E.C. adopts mks system of units," *Elec. Eng.* **54**, 1373 (1935); W. M. Hall, "The formation of systems of units," *J. Franklin Inst.* **225**, 197 (1938).

**William Harley Barber**

## Recipient of the 1947 Oersted Medal for Notable Contributions to the Teaching of Physics

*The American Association of Physics Teachers has made to William Harley Barber, Emeritus Professor of Physics, Ripon College, the twelfth of its annual awards for notable contributions to the teaching of physics. The addresses of recommendation and presentation were made by Dr. R. C. Gibbs, Chairman of the Committee on Awards, Professor Paul Kirkpatrick, President of the Association, and Professor J. W. Buchta, Vice President of the Association, in a ceremony held in the Kent Theater, University of Chicago, on December 30, 1947, during the seventeenth annual meeting.*

### ADDRESS OF CITATION BY R. C. GIBBS, CHAIRMAN OF THE COMMITTEE ON AWARDS

RECOGNITION of personal worth or achievement functions at its best in a free society. Amid such surroundings, an organization such as this Association can act on its own initiative and, without permission from any higher authority save that of public sanction of conformity with good taste, can establish an award for meritorious accomplishments of any type it wishes and can prescribe whatever procedures it desires for the selection of recipients of such awards.

Thus, the American Association of Physics Teachers undertakes annually to bestow a medal, designated as the Oersted Medal, upon a devotee in its own special realm of activity, namely, that of teaching. As a basis for this award it has specified only a single criterion—expressed in the phrase "notable contributions to the teaching of physics"—which in the last analysis is the primary objective to which this Association itself is dedicated.

The criterion for this award is simple and without qualifying restrictions. It calls for evidence of far more than average performance, in fact, for a contribution that lies outside of and beyond a mere line-of-duty activity. The territory within which such contributions may become effective is broad. They may be made in the actual teaching or in connection with the teaching of physics in the classroom or in the laboratory to students in the liberal arts course who at the time may have no intention of specializing in the subject, or to students in a technical curriculum for whom the subject is largely a requirement, or to students for whom the subject is an undergraduate major, or to students at the graduate level and beyond.

The particular quality of the contribution that weighs heavily in making it "notable" is an element of inspiration, of stimulation wherein the student becomes an active participant in the process, both mentally and emotionally; that is,

the student is induced to do some creative thinking that is permeated with enthusiasm.

Into this generalized three-dimensional picture of where, wherein, and how, which I have attempted to sketch, I should like now to project some of the attributes of PROFESSOR BARBER that led your Committee to recommend him for this award. In the $x,y$-plane we found him functioning in the classroom, in the laboratory, at the lecture table, and also as an administrator. He has worked with students at the undergraduate level, in introductory courses and in courses preparatory to graduate study. He has done his share in committee service on the ground floor, a type of academic activity that is both important and often difficult to escape.

As Dean for many years, and as Acting President for a period, he has done much to shape the policies of Ripon College. As an executive officer he has shown, according to the testimony of his colleagues on the campus, "a rare capacity for engendering and extending a broadly cooperative spirit." Thus, his contributions to teaching have been conceived and realized in many of the disciplines that contribute to training in liberal arts at Ripon.

But it is chiefly along the $z$-axis in our picture that PROFESSOR BARBER has erected monuments of accomplishment in recognition of which he has been chosen to receive an award from this Association. Arising from an area labeled "physics in a Liberal Arts College," in our basic $x,y$-plane, two enduring towers, with an interconnecting passageway at the lower level, stand out as revealing symbols of influences wrought by his inspiring efforts. These two towers represent notable contributions to the teaching of physics. The first has a broad base and is capped by an oval dome into which students were drawn from every corner of the campus by courses in introductory physics which, according to student testimonies, were thorough, exacting, permeated with a catching enthusiasm for the subject and filled with habit-forming experiences that carried over into other subjects and even into after-college activities. Very few who majored in physics at Ripon had expected originally to do so, but these introductory courses were effective even for some who later followed other professions. But the initial suggestion to major always came from the student, never from the teacher. Few ever graduated from Ripon during this period without taking at least the first-year course in physics.

The second tower in our picture is connected to the first by a passageway along which there was ever a retarding potential in the form of a reputation that ahead were stiff courses, precise measurements, painstaking reports, seminars where students played an active part, and a "no welcome" sign for the shiftless or carefree student. All who climbed up the stairs of a physics major in this tower came to know PROFESSOR BARBER personally and to share in his warm friendship. His sense of humor and his appreciation of human values, coupled with his irresistable enthusiasm for the subject under discussion, helped to instill a spirit of romance and adventure into even the more difficult and involved problems. High standards of performance were expected as a matter of course, and the students were expected to do their share of the thinking and not merely to listen and be entertained. But they liked it that way; at least, so they have said.

Many who climbed to the top of this tower, which rose high above its neighboring structures, caught the vista of higher peaks in the distance and knew, from the many references made by PROFESSOR BARBER to recent research activities, something of the opportunities to which graduate study might lead them. The relatively large ratio of the number of students who after completing a physics major under PROFESSOR BARBER went on to the doctorate, to the total number of Ripon graduates taken over the 40 years of his active service at that institution, is a striking example of a notable contribution to the teaching of physics. As one of his students who followed this path and is now a director of research in a large industrial laboratory has said, "He chose to do research in physics by creating research physicists, and their many publications redound largely to his zeal and inspiration."

*Mr. President, it is my privilege and pleasure, on behalf of the Committee on Awards, to present* PROFESSOR WILLIAM HARLEY BARBER *for receipt of the twelfth Oersted Medal, awarded annually by this Association "for notable contributions to the teaching of physics."*

# Forty Years of Physics at Ripon College

William Harley Barber
*Ripon College, Ripon, Wisconsin*

CONSIDERABLE publicity has recently been given to the fact that the small colleges of the United States have produced an exceptionally large number of the nation's physicists. The report[1] of Dr. Oswald Blackwood, of the University of Pittsburgh, on "Undergraduate origins of American physicists" placed Ripon College near the top of the list of colleges that had graduates "listed in *American Men of Science* (1938) as physicists, astrophysicists and mathematical physicists who received bachelor's degrees after 1919 from colleges and universities in the United States."

Therefore, it may be of interest to know how the physics department at Ripon College was conducted during the past 40 years. When President R. C. Hughes called me to be Professor of Physics at Ripon College in the summer of 1906, I was at the National Bureau of Standards as a laboratory assistant. Dr. S. W. Stratton, the director, objected strenuously to my leaving. I promised him that, if he released me, I would send him men who would do much more in research than I alone could possibly do. This promise has been fulfilled many times over. As outstanding students came along, it was suggested that they take the Civil Service examination in their senior year for admission to the Bureau of Standards, and many of them passed and were appointed. Ripon College has had a good representation there ever since. Thus I began my career as a college teacher of physics with the ambition to prepare my students so that they could become recognized physicists.

Up to one year before I came to Ripon College, there had been no department of physics. A general course in "natural philosophy" had been given in alternate years by the department of chemistry. In organizing the department, I was aided and advised by that great teacher, Dr. Robert A. Millikan. He gave me the mimeographed copies of what later became his textbooks in general physics as presented at the University of Chicago. This course emphasized the intimate connection between lectures and laboratory work and covered thoroughly a few fundamental principles which could most effectively be presented in connection with the laboratory work. A complete set of laboratory equipment for this course was provided by the College.

Two lectures and three two-hour laboratory periods were given each week for five hours credit. Twenty-minute quizzes were given at the beginning of each laboratory period. These covered definitions and development of equations considered in the previous lectures, as well as problems previously assigned. Close personal supervision was given in the laboratory to see that the apparatus was always in good working order and to help anyone who might be having difficulty of any kind. Duplicate data sheets of observations and calculations were kept by each student and when completed were signed and one copy filed away for reference. The student was then required to write up a complete report of the experiment, including the development of all equations used and a statement of the physical principles involved. These reports served as excellent English exercises, for they were required to be concise and accurate statements of what had been done. Many of the students had these reports bound in book form and use them today as occasion arises.

Furthermore, a special effort was made to teach physics as a science of exact measurement. The apparatus used was designed so as to give accurate results in the hands of average students. In almost every case, the result sought was found by two distinct methods and the results compared.

The course in general physics just described was given in the sophomore year and was open to all in the college who wanted a course in physics, whether for pre-engineering, premedicine, pre-agriculture or for meeting the science requirement for graduation. Many became interested and majored in physics; many went on for higher degrees in physics.

[1] O. Blackwood, *Am. J. Physics* 12, 149 (1944).

The advanced courses in physics at Ripon College were *Heat and thermodynamics* and *Physical optics* in the junior year, and *Electricity and magnetism* and *Modern physics* in the senior year. These were presented in much the same way as the first course except that three lectures or recitations and two two-hour laboratory periods were involved.

Barton, in the Introduction to *A text book on heat*,[2] outlines the scientific method as follows:

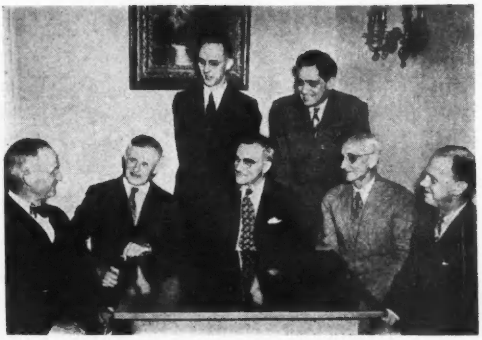
> The first and fundamental step is to ascertain the *facts* in connection with the problem under investigation, for Science recognises no authority other than Nature. The next step is to *classify the facts* in order that their significance may be the better appreciated; this involves ideally their expression in the form of a mathematical equation. The third and essential step is the formulation of a *theory* to explain the facts, this being the really outstanding part of the work, because Science is emphatically not a catalogue of facts but an attempt to fit them into a rational scheme. Finally it is demanded of a theory that it shall be capable of experimental verification and *shall lead to a search for new facts*, and so the journey is continued ever onwards into new realms of knowledge.

Physics is not an easy subject; to paraphrase an old saying, there is no royal road to physics. At Ripon College, we have required trigonometry as prerequisite to general physics, calculus as prerequisite to heat and optics, and differential equations as prerequisite to electricity and magnetism. If, as has been stated many times, differential equations are the language of physics, these requirements are justified. The closest cooperation between the physics and mathematics departments at Ripon College has existed for the past 40 years.

All physics majors were expected to take chemistry, including organic and physical, and many of them graduated with majors in physics, mathematics and chemistry. There was no segregation of these three departments in the minds of the students, for they all dealt with one indivisible nature. To a large degree, these departments approximated the group idea that is now used in some colleges in lieu of departments. Other courses recommended for physics majors included German, French, American history, public speaking or debating, economics and philosophy. Some of those now in high positions in industry have told me that their course in public speaking has been of particular value to them. The graduates with physics majors, therefore, came out with a well-rounded education and an appreciation of the scientific method which fitted them for many fields other than graduate work in physics.

Ripon College grants no degrees beyond that of Bachelor of Arts. It is held that those wishing to do graduate work should go to the universities for their master's and doctor's degrees. This is especially true for those seeking advanced degrees in science, for the colleges are not equipped generally with the expensive laboratory apparatus needed for present-day graduate work in the sciences. Throughout the years, teaching and research assistantships in many of the leading

Physics laboratory, Ripon College.



Professor Barber (left) and several former students: W. F. Meggers, National Bureau of Standards; W. W. Mutch, Knox College; Neil Morgan, Wisconsin Telephone Company; L. P. Goodrich, Superintendent of Milwaukee Schools; Austin Ely, Kimberly-Clark Corporation; J. H. Dillon, Textile Research Institute.

[2] A. W. Barton, *A textbook on heat* (Longmans, Green, 1933), p. xi.

universities in the United States have been available for our physics graduates, particularly for those who have served as senior laboratory assistants here. These are the men who ultimately have been listed in *American Men of Science*.

In my opinion, the fact that so many physicists have come out of Ripon College during the past 40 years has been due to the practice of selecting the outstanding senior physics student each year as a laboratory assistant in physics. This has been a one-man department all these years, not from necessity, but from choice. In 1906, we had a college enrolment of 202 and in 1946, when I retired, 530. In the later years an assistant professor might have been added, but this would have eliminated the opportunity for some exceptionally gifted student to serve as an assistant and thereby become interested in going on into graduate work in physics. These assistantships have been keenly sought after, and most of those who have gone on for their doctor's degree in physics have served in that capacity. One of them wrote to me as follows:

> Judging from my experience as a senior laboratory assistant at Ripon College, a good student cannot obtain better training in any other fashion, and it is certainly a desirable thing to have. At present I am teaching engineering students here at the university, which means that I will have no physics majors, thus robbing me of the satisfaction of having a few good physicists leave every year who might later remember me as having given them their fundamental training. I am quite sure that it is that sort of thing which has given you your greatest pleasure in your work of teaching, and it is certainly something I do not want to miss.

This letter is one of the compensations that has come to me as a teacher of physics for 40 years. This young man turned down several opportunities to go into industrial research, selecting, at a considerable financial sacrifice, the life of a physics teacher. I prophesy that when the opportunity arrives he will be the teacher to turn out many an American physicist of the future.

In conclusion, I wish to pay my respects to my first physics professor, Dr. B. W. Snow, of the University of Wisconsin, who inspired me to become a teacher of physics, to former President Silas Evans, of Ripon College, who always gave me freedom to conduct my department in my own way, and to Dr. Robert A. Millikan, who guided me in organizing and conducting the department at Ripon College in such a manner that it has culminated in my being named the 1947 recipient of the Oersted Medal of the American Association of Physics Teachers.

## The Social Implications of Science

ROBERT WEIL
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> *Socrates:* "If then each is ONE, and both together make TWO, the mind will conclude that the two are separable. For if they were inseparable, it could only conclude that they are ONE, not TWO."
>
> PLATO, *Republic*, Book 7.

IF the reader expects a passionate plea for the advancement of nuclear physics because of its potential benefit in the research on cancer; if he is getting ready for a rousing harangue against it because of man's inability to keep pace with his inventions; or if he awaits a saintly sermon against science in general because of its detrimental effect on religion and thus on society: he will be well advised to discontinue reading. For the foregoing are some of the social implications of applied science but not those of science proper. We think that, in failing to differentiate between the two, Bernal has chosen a wrong title for his treatise, *The Social Function of Science*. The well-being of mankind is governed by the application of scientific principles; but their behavior as individuals and members of society has been determined for thousands of years by pure science—or lack thereof. It is because we draw this line that our subject will be treated rather from a historical point of view. Then it will be understood why more importance is ascribed to Bacon than to Becquerel, and why we think that Newton is the only scientist whose work has had any social implications.

The oldest science, in all probability, is social science. People discovered that, when there were large numbers of them, an enemy could be warded off more easily than when there were few. They also found that the organization of families in groups facilitated the provision of their daily bread. It follows that military science soon became a factor which had a bearing on life. But, as far as the pure sciences are concerned, it must be conceded that, in Greece at least, they were determined by social concepts and not *vice versa.* Although astronomy, mathematics and anatomy had not been unknown to peoples older than the Greeks, interest in those branches of science had been prompted mainly in order to foretell the future. Only in isolated cases can utilitarian reasons be detected. If, then, we consider that pure science was initiated by the Greeks—apparently the first who knew what contemplation meant—it is evident that the thing in existence, a social order, would affect that which was being born.

Although democrats, the Greeks had found it profitable to divide their nation in two: masters and slaves. The latter worked principally with their hands, the former with their minds. Hence the speculative aspect of Greek philosophy, the wild scientific surmises of a mind like Aristotle's, and the nonexistence of experimental tests. Considering how despised manual labor was, it must amaze us to discover that there were artists who would produce works like the Laocoön or the Venus of Milo. On the other hand, it would be wrong to assume that no experiments had been performed at all—compare Archimedes, Galen, and others. In comparison with the mental output, however, their volume was negligible.

Yet the value of science was not underestimated. Arithmetic, plane and solid geometry, astronomy—pursued abstractly as dynamics—they all belong to the syllabus that Plato proposed for the education of the Perfect Citizen. And when Glaucon eagerly approves of the suggestion that astronomy should be made the third study, he points out its importance for the farmer and the sailor; Socrates laughs, and this laughter makes Bacon squirm. "You amuse me," Plato makes Socrates say, "you amuse me by your evident alarm lest the multitude should think that you insist upon useless studies." The study of all these branches of science contributes to the perception of the Greater Reality whose shadows only we observe, prisoners fettered to our senses though we are. This is Science for the Sake of Science, and any utilitarian attributes should be dismissed.

We have thought it necessary partly to enlarge on these scientific implications of society because of their effect on the Renaissance. From our point of view, the period between the later Greek philosophers and, shall we say, Petrarch can be ignored. It is true, discoveries and inventions were made in that interval; but science, defined as the organized accumulation of facts, made little headway.

Now the Renaissance produced an entirely new type of man. For the first time in his history, as far as we know it, man became an individual. This manifested itself also in the cosmopolitanism which appeared in those days. Already Dante, exiled from Florence, is not satisfied with the riches of Italy alone: "My country is the whole world." These words are as significant as their echo, "The world is my parish." Admittedly, the latter "world" was considerably larger than Dante's. But the step would seem to be in the right direction. At the same time, it should be remembered that, as far as science was concerned, Dante was a child of Aristotle's. In "Die mathematische Denkweise," Speiser has pointed out the intensely mathematical structure of the Divine Comedy. Even so, the Florentine's cosmogony hangs fire. But, and let there be no doubt about that, in his day Dante was as much a scientist as a poet. His present fame as the latter is, of course, due to the fact that his poetry was true and his science false. Petrarch, in turn, was a poet and a geographer. And later, Raphael founded what came to be known as the science of archaeology.

Are these remarks as irrevelant to the subject under discussion as might appear at a first superficial glance? One of the conclusions to be drawn is this: although only a few have been mentioned, the number of very great men in Italy at that time is really astounding. If we bear in mind that social advantage due to birth was nonexistent, that, with the exception of the later generations of the great families, the foremost intellects were upstarts, and that honor was given

where it was due, we must conclude that the general level of intelligence of the "common multitude" was, in all probability, very high. Other countries have on many occasions experienced flourishing periods. But in what all-round way have the Troubadours and Minnesänger contributed to the general advancement of culture? The fullness of life in Italy in the times of the Renaissance may be justly ascribed to the simultaneous impact of art and science. The former was mainly restricted to the fine arts, although the age produced a Palestrina. However, while the latter personified only a beginning, the fine arts reached great degrees of perfection. Granted, those days also produced a Cesare Borgia. But in spite of him, it would appear that a possibly fortuitous mixture of those two ingredients—the intellectual and the emotional—produced a flower the like of which has not been seen before or since.

Not a little responsible for this great progress of European culture was the Church. Popes like Nicholas V or Pius II encouraged the new trend of humanism and scholasticism to the best of their ability. Only characters like Alexander VI brought the whole of the edifice to ruin. It is interesting to compare those days with our own. The Church not only encouraged the perpetual excavations and breathless hunts after manuscripts across half of Europe; it also financed them, and numerous cardinals and many a pope took a most active interest in all these pursuits. We are ready to accept the argument that, if the clerical dignitaries acted as Maecenases, and commissioned great painters and renowned architects to work for the glorification of our Lord, they may have hoped that some of the reflected light would dazzle the historian. They may have wanted to hide the rising spiritual squalor: their generosity may have been based on selfishness. But what glory would they think could accrue to them from the excavation of a Roman pillar or the discovery of Terence's comedies? Today we have a similar encouragement on the part of the Government. Scientific research is being financed by public means at last. Whether this is done generously or not is of no consequence in the present comparison. What is relevant is the motive behind this action of theirs and how the allocation of funds compares with that for the Arts Council. We think that the latter receives something less than four hundred thousand pounds in the current year, while the expenditure on research runs into millions. The reader will perceive the reason for this discrepancy: material benefit may result from scientific research, but not from the activities of the Arts Council. No material advantage was derived from the discovery of long-forgotten manuscripts or damaged statues, but man was, in most cases, a worthier being.

The new knowledge managed to cross the Alps. It produced a Spinoza as it produced a Luther, and culminated formally in the pansophism of Comenius. But when it crossed the Channel, the reaction, which had commenced already in Italy, found its chief exponent in England's faithless High Chancellor and the first of her modern philosophers.

We have seen that the avowed object of Greek knowledge was the perception of the Reality behind the appearances as presented by our senses. In extreme forms this led to the utter disregard of material welfare, as is shown by the Stoics. Here we may be allowed to mention Diogenes. This austere pursuit of the spiritual was accepted by the early Christians, and later Diogenes turns into St. Francis of Assisi. To complete the symmetry: when the Renaissance period tails off into the baroque, it is the Church that first degenerates, valuing material fortunes above all others, and philosophy comes afterwards.

This is manifested by Bacon's work. In effect, he says, science is of no value unless it is conducive to the improvement of man's material position. If Plato values arithmetic for its own sake, Bacon will have it only as a tool which is employed in physical researches. If mathematics—probably only algebra, since geometry had practical applications—would lead us to the knowledge of eternal truths with Plato, with Bacon mathematical science was only an appendage to other sciences. It is "the handmaid of natural philosophy" and "ought to demean herself as such." As Macaulay would have it, "Bacon fixed his eye on a mark which was placed on the earth, and within bow-shot, and hit it in the white. The philosophy of Plato began in words and ended in words. . . ."

Now we are beginning to understand why the

Government should finance research. Bacon himself says, more or less, that an inquiry after Truth is worthless unless more coal is produced with a smaller effort as a result of the discovery of Truth. Not satisfied with stressing the necessity of keeping an eye on its utilitarian value, he decries natural philosophy as a waste of time. We cannot blame him for his attitude—he completely ignored moral philosophy—because he personifies a reaction, and a reaction is rarely associated with the golden mean. What we are inclined to deplore is that, today, nearly 350 years after Bacon's time, the same attitude should prevail. But to this point we shall return later. For the moment it should be stressed that Bacon's philosophy is the first, and probably the farthest reaching of the modern implications of science. It did not remain confined to the spheres of science: but how the modern mercenary mind is to be accounted for is not within the scope of this essay.

Like all great philosophies, Bacon's did not make itself felt immediately. So we find Descartes trying to construct a system of abstract thought by using the most elementary property of the Euclidian triangle, namely, the fact that the sum of its internal angles is equal to 180°.

All churches appear to be gerontocracies. It is therefore safe to assume that, when Galileo was made to recant, the gulf between learning and the orthodox dogma imposed by the Roman Church must have been widening for a considerable time if that body thought it necessary to stage a trial. There is no foundation, in either Testament, for the belief that the sun circles round the earth. Yet the Papal Curia did not abandon that belief officially until 1827! This illustrates how perverse the Church had become: established to guard the interests of our Lord, it invoked Him to guard its own. By a transference of the argument it considered a questioning of its authority to be a doubt in His. So it happened that this Church which, only two centuries previous to that time, had encouraged and guided all intellectual pursuits became the seat of scholastic reaction. Automatically Reason became a heretic because it would not always tally with Faith. And Faith was what the Curia said it was.

But narrow-mindedness was not, as many would like us to believe, a privilege of the Church of Rome or of those times. It is one of the jokes of history that Newton's formulation of the expression

*Force = Time Rate of Change of Momentum*

should have given rise to Lenin's materialism. It is a joke because Newton was no godless man. The connection is not as far-fetched as would seem. Up to the close of the 19th century, Newton's mechanics enabled people to explain everything to which it could be applied. If natural phenomena—like gravity—could be accounted for as easily as that, why not other occurrences as well? The fact that physics could be understood removed the awe in which everything unknown is shrouded, and rationalism was born. This led immediately to Voltaire's atheism. While the hopeless finances of France provided the trigger for the storming of the Bastille, and Tom Paine and the American War of Independence may have been the rifle along which the charge shot when it was fired, it may not be wrong to suggest that the French Encyclopedia, directed by Rousseau, Diderot, and others, was a guiding factor when the aim of the shot was discussed. If reason can explain everything, do we need any gods? So Napoleon takes the crown from the Papal hands and crowns himself Emperor.

It is interesting to note here why "La république n'a pas besoin des savants," as was said when Lavoisier was sentenced to die. Today a scientist's ability is rated above everything, including his character. Thus England found herself compelled, for economic reasons, to "import" enemy scientists, and the Germans employed some Jewish scientists if they needed their services very badly. One hundred and fifty years ago, science was only getting into its stride. Its military value was negligible—although Napoleon was far-sighted enough to take several scientists to Egypt—and the mob objected to the scientist because, paradoxically, a certain type of deism was associated with him.

It is easy to see how the Mendelian theory of heredity and Darwin's theory of evolution—nearly solving the riddle of Man—contribute to the picture of omnipotent Reason. Add *Das Kapital* of Marx, and the 20th century is under-

stood. "Provide for material needs"—Bacon?—"and the rest will follow," thus runs a modern creed. Lenin will explain thought in terms of electronic processes, or, rather more carefully, he says that, one day, it will be thus explained.

So we see how deeply science has become engrained in society. Bacon broke away from the intellectual aristocracy, encouraged experimental study, and the latter succeeded beyond all expectations. But the clouds of the coming misunderstanding could not be dispelled. Rationalism and Dialectic Materialism appeared on the scene. Kelvin's vision of being able to understand anything provided it could be turned into a machine finds its counterpart in social science in the rejection of anything spiritual. It should be noted that Kelvin was referring to physical phenomena only.

The unequivocal success of Bacon's method and Newton's principles in the realm of the physical world, the interpretation of this success by well-meaning but incapable journalists, and the realization of the magnitude of atomic forces has led to the heresy that science is omnipotent. Trained scientists know that this is not so, and that it is even fundamentally impossible. But the general public are still pseudo-Kelvinists. The proposition about the omnipotence of science will now be examined in closer detail.

The argument will be divided in two: it will first be shown that, when it comes to the point, science is as little potent as ever. Secondly, we shall deliberate in what way the outlook of the scientist might have to be modified so as to reëstablish an intellectual balance of power, and to offset the apparently evil influence that science has had on society in recent times.

Since some use will be made of its implications, it is desirable to give a brief account of Heisenberg's principle of indeterminacy. The symbolic representation is very simple, but serves no useful purpose here, and is therefore omitted. The following description is partly based on Sir Charles Darwin's ideas as expressed in *The New Conception Of Matter*.

Electrons have the property that they move when struck by a beam of light. This has been confirmed by numerous experiments. These electrons are so small that they cannot be seen at all; but, for the sake of argument, it is assumed that a super-microscope has been set up, and that the magnification is such as to give us a chance of seeing one electron. We further assume that we have succeeded in fixing an electron to the slide, and that it waits there until we can see it. To see it we need light. Therefore we direct a beam of light at the electron, and off it goes. However frequently we may repeat this experiment, the electron flies away. But we are not going to surrender. We shall enclose the electron, and then it cannot escape when we direct the beam at it. Yet even so it will jump about in the enclosure, and its position will be known only by the limits of the enclosure. If the latter is reasonably large we shall be able to measure the product of its mass and velocity, its momentum, to a great degree of accuracy. Someone clever will suggest that the enclosure should be reduced until it has the size of the electron: then we could still measure the momentum, and we would know where the electron was. This is precisely where the clever one is spited by the electron: when the size of the enclosure compares with that of the electron, the latter simply cannot be contained and breaks out. Also how could we get the requisite apparatus into a very small enclosure? We see, therefore, that we can either know the position of an electron to a great degree of accuracy or we can know its momentum equally well; but *we cannot know both to a very high degree of accuracy.*

This explanation is very inadequate. But it may serve to indicate that an element of the unknowable has crept in somewhere. This is not due to a logical error or a fault in the mathematics which state the italicized phrase in a formal manner, but may be ascribed to the nature of things.

A wise man would leave it at that. However, the unrelenting propaganda of some of the clergy—for instance, the Bishop of Liverpool, 1930—may have caused the scientist a bad conscience. His materialism and the precipitation of Doomsday were said to be the result of his labors, and these were ungodly. Now Heisenberg had given the scientist a chance: if not everything can be determined, then I may have a measure of free will; if I have a measure of free will, not everything can be matter; if everything is not matter, I may find room for a god in this world; if I can find room for a god, I can also find room for God; if I can find room for God, I need not quarrel with the parson. The scientist and the clergyman embrace, issue a joint communiqué to the effect that, after all, science does not contradict religion and *vice versa*, and they live happily ever after.

The fallacy occurs, as usual, in the assumption that there is a mathematical relationship between electronic processes and the human mind. This

may be the case, but until we see the equation we need not believe in it. The onus of the proof lies with the prosecutor. We are quite satisfied to find a mathematical expression which states that everything cannot be known. It should be remembered that this is only the first of its kind. As far as we are concerned the scientist and the clergyman can live happily without embracing. That will be quite easy provided they do not poach on each other's territory. In his eminent little essay, "What Is Life?" Schrödinger tries to prove immortality! We fail to see why matters of faith should be subjected to the scrutinizing eye of Reason, and why, if unusual occurrences become understood, their value as miracles should be reduced or destroyed.

There is no cause for despondency on account of Heisenberg's principle. It is always an achievement to get to know one's limitations. Physical science has discovered one of them, and should gladly proclaim the fact lest the false impression which people have of its supposed omnipotence should lead to further complications. This omnipotence and omniscience ascribed to the scientist wrongly and grudgingly has often been complained of. In the popular mind it is assuming dangerous proportions: the other day we discovered an Atomic Restaurant, and passed by a van that proudly exhibited the firm's name—Atomic Carpet Cleaners. It took about 150 years until the broad masses understood the implications of Newton's second law. Improved methods of education should be able to drive home the implications—the true implications—of Heisenberg's principle of indeterminacy in a shorter time. Since the latter is more abstruse, the job may be more difficult. It will be more difficult for, after all, if the scientist is omnipotent, I can be, and gods have a way of resenting dethronement.

Such a strategic withdrawal—that is what it will seem to the ignorant—may be of some use in doing away with a misunderstanding. But removing something negative does not necessarily produce something positive. Our object should be to prevent future philosophical catastrophes and, at the same time, to ensure that the genius of science can flourish in conjunction with the others which lead to a deeper understanding of our world. Bernal writes: "Science will clearly be the characteristic feature of the third stage of humanity." The wary eye discerns in this another attempt at establishing the hegemony of science. This should be avoided. Aristotelian science has not survived, Leonardo's art has. We expressed our belief that life was at its fullest when the various pursuits of the human mind resided simultaneously and not specifically in one being. Two or three hundred years ago, the scientist, the craftsman, the artist were all one. Then they split apart, and specialization set in with its very mixed blessings. There was a time when Pepys or Wren could be Fellows of the Royal Society. Today the honor is conferred principally on scientists; only when the Fellows wish to honor themselves do they elect a nonscientist.

We hold that the cultural outlook of Everyman should be widened to a very great extent. It is futile for the young artist to argue that he is "doing science" when he studies anatomy or pigments or architecture. It is equally futile to say that the budding scientist is showing any interest in art when he is holding his partner's hand at a cinema performance of "Henry V." A broad culture can be obtained primarily by a broad education. And that is where, socially, science can get to the root of the problem. That is where modern technical colleges, combined as they often are with schools of art, can outspan the more rigid universities. But a generous, visionary, daring, and unflinching outlook is required above all. Such a project cannot be left in the hands of institutions like UNESCO while their prime occupation is the eradication of illiteracy. It would appear that it cannot be left with any national body because, in spite of its renaissance touch, the scheme is advanced. A few individuals who have the opportunity of putting the plan into practice should do so and stand or fall with it. We must plead for the replacement of the old Greek dualism—the harmony of body and mind—by the trinity of religion, art, and science. The balanced application of these may perhaps lead us to a new scholasticism, which, we trust, will give the unity to our knowledge and our conception of the universe that their nature demands. And as a by-product, we may discover the road that leads to less squalor, less disease, and less misery.

# Reproductions of Prints, Drawings and Paintings of Interest in the History of Physics

## 36. Raphael's "School of Athens"

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THE "School of Athens" is one of the outstanding achievements of that great and versatile artist, Raphael Sanzio (1483–1530). This noble fresco, painted in the years 1510–1511, still adorns one wall of the *Stanza della Segnatura* (Papal Signature Room) of the Vatican. It depicts with consummate skill an assembly of the great philosophers and men of science of ancient Greece arranged, posed and executed in such a way as to display not only the highest artistic genius but also a very considerable knowledge of the history of philosophy and science.

In the center of the picture (Plate 1), at the top of the flight of steps, stand Plato, the idealist, and Aristotle, the realist, flanked by philosophers of their respective schools, the idealists on the left, the realists on the right. On the lower level, the two corresponding schools of scientific thought are portrayed, on the left the Pythagoreans who believed and taught that pure reason alone can reveal ultimate truth, and on the right the Archimedians who held that reliable knowledge of the physical universe cannot be attained without observation and experiment. The whole

Plate 1. Raphael's "School of Athens."

composition symbolizes the essential harmony between philosophy and science, between idealism and realism, between analysis and empiricism, between reason and experiment.

The popular explanations of this great painting that are encountered in travelers' guides[1] and in descriptions of the art treasures of the Vatican are that RAPHAEL has symbolized in it the course of development of Greek philosophy and that it is possible to assign definite names to all the individual figures. Such explanations go much further than is warranted;[2] nevertheless it is evident that the figure at PLATO's right, talking with gestures to the youth in armor, represents SOCRATES, that the man lying on the steps is DIOGENES, that the central figures of the two lower groups are probably PYTHAGORAS (left) and ARCHIMEDES (right), and that the individual in the lower right-hand corner wearing a crown and carrying a globe in his hand is PTOLEMY, for common tradition confused the astronomer with the Egyptian king of the same name.

The skill with which RAPHAEL has "clad abstract ideas in forms of life and beauty" is nowhere better displayed than in the group of Pythagoreans at the lower left (Plate 2 is an enlargement of this group). PYTHAGORAS himself is seated and is writing his discoveries about harmony and numbers in a book held on one knee. The oriental figure peering over his shoulder probably symbolizes the effect of the mysticism of the East upon his thinking. The boy holding before his master the number diagrams that he has drawn upon a board may well symbolize the hope that the seemingly "infinite complexity of nature is really as simple as a child's arithmetic." That Pythagorean science was broad, human and universal in its appeal is made clear by the various ages, sexes and nationalities represented in the group (the head of the young woman behind the oriental figure may have been inserted for this reason or to indicate the fact that women were admitted to Pythagoras' lectures). The two figures that lead the eye up to the higher level, one an ethereally beautiful youth, the other a man in the prime of his years, may well symbolize and contrast the religious mysticism of Pythagorean philosophy with the more lasting mathematical and scientific achievements which, it may be supposed, are demonstrated in the book the mature man holds on his thigh and calls to the attention of the group. The solitary individual seated still further to the right and resting his elbow upon a marble block probably does not

PLATE 2. Enlargement of the lower left-hand portion of Plate 1.

[1] See, for example, M. Starke, *Information and directions for travellers* (London, ed. 8, 1832), or the extract in Appendix I of P. E. Mottelay's *Biographical history of electricity and magnetism* (London, 1922), pp. 542–544.

[2] See Trendelenburg, *Ueber Rafael's Schule von Athen* (Berlin, 1843), and A. Richter, *Ueber Rafael's Schule von Athen* (Heidelberg, 1882).

PLATE 3. Enlargement of the lower right-hand portion of Plate 1.

belong to the group proper and is usually identified as DEMOCRITUS because he is booted in the manner of his countrymen, the Abderites.

The spirit and procedures that characterize teaching and learning at their best are beautifully depicted throughout the picture and give it its modern name, the "School of Athens." Thus, in the group at the lower right-hand corner (shown enlarged in Plate 3), ARCHIMEDES bends down and draws geometric diagrams on a blackboard. Four pupils surround him and listen attentively to his demonstration. In them RAPHAEL has pictured with unexcelled skill the degrees of understanding and the process of gradual mastery of the subject matter. One boy kneels on the earth and follows with close attention the hand of the teacher; he imitates the master's motions, but he does not understand. Leaning over him, stands another boy; his face, as well as the motions of his hands, indicates that he not only sees, but that he understands what he sees. A third boy with uplifted face, having mastered the demonstration, imparts it to the fourth boy. And the face of the fourth reflects the joy of achievement and complete understanding; he is able to conceive of the far-reaching consequences of the theorem that has been proved.

This magnificient painting is surely worthy of reproduction and study at a time like this when physics and philosophy, originally one but separated since the Renaissance, are again converging and merging in the deeper analyses and the newer developments of quantum mechanics and relativity, and when the spiritual and social values and responsibilities of science need greater emphasis than they have received in the past.

## Boners

From an examination paper in a course on *Physical Science:* "The force of universal gravitation equals gravity times the mass of the earth times the mass of the sun divided by the radius of the earth squared."

[*Contributed by Richard M. Sutton, Haverford College.*]

# NOTES AND DISCUSSION

## Thermal Insulation of Building Materials

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IT is more or less generally assumed that the quality of an insulating material is determined by the value of its thermal conductivity; that is, the problem is assumed to be one of steady heat flow from a region of fixed high temperature to one of fixed low temperature. In winter this is probably a good assumption, for then the temperature inside a house, for example, remains substantially constant at a value higher than the outside temperature and there is an approximately constant flow of heat from the inside to the outside.

The situation is somewhat different in summer. During the hottest weather, the outside temperature rises above the inside temperature during the day and drops below it at night. Heat is alternately flowing in during the day and out during the night, and the wall itself is alternately being heated and cooled. The problem is one of variable heat flow and obeys Fourier's equation,

$$\nabla^2\theta=(1/h^2)\partial\theta/\partial t,$$

where $\theta$ is the temperature at a point within the wall, $h[\equiv(k/c\rho)^{\frac{1}{2}}]$, the coefficient of diffusivity, $k$ the thermal conductivity, $c$, the specific heat and $\rho$, the density.

To get a solution of this equation, it is assumed that the outside temperature is given by

$$\theta=\theta_0\sin(2\pi/T)t,$$

where here $\theta$ is the temperature of the outside surface of the wall, $\theta_0$ is half the daily variation in temperature of the outside of the wall and $T$ is 1 day. It is also assumed that each portion of the wall has the same temperature distribution that it would have if it were of infinite thickness, so that the temperature of the back side of a given portion of the wall is taken to be the same as it would be at a like distance into a wall of the same material of infinite thickness. On the basis of these assumptions, if $\theta_{max}$ is half of the daily variation in temperature of the inside of the wall, a solution of Fourier's equation results in the expression

$$\theta_{max}=\theta_0 e^{-S(\pi/T)^{\frac{1}{2}}},$$

where $S$ is the sum of the values of $\Delta x/h$ for each part of the wall and $\Delta x$ is the thickness of each part.

TABLE I. Rate of conduction of heat and value of $\theta_{max}$ for typical walls.

| | Wall | $Q$ ($10^6$ cal/sec cm$^2$ deg C) | $\theta_{max}$ (deg C) |
|---|---|---|---|
| *A* | 8-in. brick, ¾-in. plaster | 7.0 | 1.8 |
| *B* | 12-in. brick, ¾-in. plaster | 4.8 | 0.7 |
| *C* | 8-in. brick, 3-in. rock-wool, ¾-in. plaster | 1.0 | 0.9 |
| *D* | 1¾-in. wood, 3-in. rock-wool, 1-in. plaster | 1.0 | 2.4 |
| *E* | 1-in. wood | 10.8 | 6.7 |

TABLE II. Values of the constants used in calculating Table I.

| Material | $k$ ($10^3$ cal/sec cm deg C) | $c$ (cal/gm deg C) | $\rho$ (gm/cm$^3$) | $h$ ($10^3$ cm/sec) |
|---|---|---|---|---|
| Brick | 1.5 | 0.20 | 1.5 | 7.0 |
| Plaster | 1.2 | .18 | 1.2 | 7.4 |
| Rockwool | 0.09 | .20 | 0.09 | 7.0 |
| Wood | .27 | .42 | .5 | 3.6 |

The insulating value of a wall in summer is indicated by the value of $\theta_{max}$. It should be noted that $\theta_{max}$ must be added to the mean temperature to get the actual maximum temperature and also that $\theta_{max}$ is not necessarily the room temperature, because all of the walls of a room would not be at the same temperature and, further, because air convection has been neglected.

To illustrate the use of the foregoing theory, calculations were made for five typical walls. The rate of conduction of heat $Q$ and the values of $\theta_{max}$ are given in Table I. The daily variation in temperature of the outside wall was assumed to be 20°C. The values of the constants used are shown in Table II.

Thus, Table I shows that while wall *A* conducts seven times as much heat in winter as *D*, yet in summer the inside temperature of *A* is 0.6 deg C lower than that of *D*.

Table II shows that the diffusivity of rockwool and of brick are the same. This would indicate that rockwool possesses no particular advantage over brick for summer insulation. This point is also brought out in Table I through a comparison of walls *B* and *C*. The only difference between these two walls is that 4 in. of brick for wall *B* has been replaced by 3 in. of rockwool for wall *C*. Table I shows that wall *B* conducts five times as much heat as wall *C*, yet is slightly cooler in summer.

The effect of adding more and more insulation to a wall would be to bring $\theta_{max}$ nearer and nearer to zero. Thus the addition of further insulation to walls *B* and *C* would cause a lowering of the inside temperature by less than 1 deg C.

The assumptions on which the foregoing calculations were based are not strictly valid, but the problem becomes very difficult if they are not made. Experiments were started last summer to check this theory. Little progress was made owing to the fact that the hottest part of the

TABLE III. Experimental results for a brick wall 8 in. thick, ¾-in. plaster.

| Date 1947 | Average temperature of outside (deg C) | $\theta_0$ (deg C) | $\theta_{max}$, observed (deg C) | $\theta_{max}$, calculated (deg C) |
|---|---|---|---|---|
| Aug. 26 | 34.7 | 9.2 | 2.0 | 1.7 |
| 27 | 39.0 | 7.5 | 1.7 | 1.4 |
| 28 | 30.5 | 7.5 | 2.0 | 1.4 |
| 29 | 30.0 | 8.0 | 2.0 | 1.4 |
| 30 | 39.0 | 8.0 | 2.5 | 1.4 |
| Sept. 1 | 27.5 | 5.5 | 1.2 | 1.0 |

summer had already past. Table III gives a brief summary of the data that were obtained. It is hoped that it will be possible to carry the experiments further next summer.

## Ranking Ballot More Democratic

Edward M. Little
*Naval Ordnance Laboratory, Silver Spring 19, Maryland*

THIS is in response to the recent note by J. D. Elder [*Am. J. Physics* **15**, 429 (1947)] regarding nomination and election of officers. Some very good ideas are presented, to which I should like to add a few. This subject has interested me for many years.

In state primary elections where there are several candidates for governor, in final elections where there are many parties as in France or sometimes in this country, or whenever there are more than two options (either candidates or propositions), the usual election is far from being truly democratic. When each voter has only one vote, a minority group of voters may combine their votes to elect their man or proposition, if the voters in the majority group scatter their single votes rather uniformly amongst the other candidates, thus wasting their votes. How can this undemocratic result be avoided?

There are several methods, but the ranking ballot seems the best of those that are not too cumbersome. It is used, for example, by the Philosophical Society of Washington. In this system, if there are, say, six candidates, each voter would mark six votes opposite his first choice of candidates, five opposite his second choice, . . . , and one opposite his last choice. (The afore-mentioned votes could all be reduced by one to save tellers' work, but it might be more confusing to some voters.) The candidate (or candidates, if there are more than one to be elected to an office) polling the largest number of votes wins. Ballots with no votes for one or more of the candidates are thrown out. One balloting is always sufficient, even when two or more are to be elected; exasperating successive ballots are avoided.

In this method nobody wastes votes, as each person's ballot contributes different amounts to the final rank of *each* candidate rather than to the final rank of only one candidate. It is true that limiting one's votes to consecutive integers can rarely represent accurately the relative worth of the candidates in the opinion of the voter; but it is better than the single-vote system and less cumbersome than other methods that might do the job better. Furthermore, the voter is more likely to attempt to learn about *all* the candidates when he has to rank them all. The winner has the highest integrated worth of all candidates, not necessarily, or even usually, the highest number of first-place votes; the former is much preferable and is more democratic, in my opinion.

Here, then, are my recommendations regarding nomination and election procedure; they incorporate some of the good ideas in Professor Elder's letter.

(1) Elect only the president, who should appoint the other officers. We elect too many officers; the president would usually know better than the rank and file of us who are the best men for secretary, treasurer, and so on.

(2) Give all members an opportunity to nominate by mail before, say, October 1. A nominating committee appointed by the president should also add enough names to bring the total number of candidates up to at least two.

(3) List the candidates' backgrounds either by reprinting the account that appears in *American Men of Science* or, if not there, by printing a statement prepared by the candidate himself.

(4) Use the ranking ballot system.

The term of the president could be either one year or two, preferably one. There would be no write-ins—there would be opportunity for such at nomination time. The vice president should not become president automatically; if he or any other officer, including the president, has been an outstanding officer in the preceding year, he would probably be elected president. The president could succeed himself, but it would be only by real accomplishment, not by "railroading." Under these conditions there would be less tendency to decline nomination, for no officer feels as complimented by a railroaded election as by a truly representative one.

## The Mousetrap Bomb: Modification $N+1$*

J. H. Manley
*Los Alamos Scientific Laboratory, Los Alamos, New Mexico*

ALTHOUGH the most urgent advance in this new field, security permitting, is undoubtedly the development of an automatic—and, of course, electronic—trap resetter, Professor Sutton's letter[1] encourages one to add other features to the mousetrap atomic bomb.

The cardboard reflector used to return corks to the traps may be replaced by a spatial array of solid objects more nearly analogous to the atoms of an actual neutron reflector. Rather than attempting to obtain an arrangement of small spheres, one can obtain good results from rods or bars arranged in the fashion of boiler tubes. A change in the spacing (always keeping it greater than cork size) and rod diameter convincingly demonstrates the desirability of a reflector composed of large atoms closely spaced, so that they have short mean free path. Replacement of the rods by thin sheets or bars introduces the effect of the angular distribution of scattering on the efficiency of the reflector; that is, it shows the role of the transport mean free path rather than the simple collision mean free path.

Since any actual reflecting substance also has a chance of capturing a neutron, small cylinders of Superstick fly paper may be appropriately distributed over the rods, though it must be confessed that this makes the operation a bit messy.

It should not be overlooked that the use of a spatially distributed reflector has introduced an observable time element into the chain reaction, namely, the time between

emission of a cork from one trap and its action on another. One must be careful that this is not interpreted as being analogous to a slow neutron reactor, but rather that the degree of criticality is related to the time constant of the system through the time between fissions.

In case one does not become discouraged at slight complexities, the rod reflector may be combined with a spatial arrangement of mousetraps in order to approach a lattice-structure pile. Need one go on?

Finally, if Professor Sutton's suggestion of increasing one's satisfaction by using rat-traps and baseballs is followed, it would seem worth while to push the project analogy to the point of including a medical section in the interest of the health of the worker.

* This paper is, in part, based on work performed at Los Alamos Scientific Laboratory of the University of California under Contract No. W-7405-eng-36 for the Atomic Energy Commission, and the information contained therein is not expected to appear elsewhere.

[1] R. M. Sutton, *Am. J. Physics* **15**, 427 (1947).

## New Members of the Association

The following persons have been made *members* or *junior members* (*J*) of the American Association of Physics Teachers since the publication of the preceding list [*Am. J. Physics* **16**, 62 (1948)].

**Avil,** Richard D., 2635 E. Thompson St., Philadelphia 25, Pa.
**Balber,** David, Clinton National Laboratory, P. O. Box W, Bldg. 703 B, Oak Ridge, Tenn.
**Berry,** Herbert W., 503 Wolcott Ave., Syracuse 4, N. Y.
**Damon,** Paul E., 702 B Pine St., Rolla, Mo.
**Dangers,** Elaine, College of Saint Teresa, Winona, Minn.
**Goberna,** Joseph R., Observatorio de Belen, P. O. Box 221, Havana, Cuba.
**Haas,** Robert D. (*J*), Western Michigan College of Education, Kalamazoo, Mich.
**Hagen,** Jack Ingval (*J*), Oregon State College, Corvallis, Ore.
**Hixson,** William F. (*J*), 802½ Knoblock St., Stillwater, Okla.
**Hopkins,** John I., Vanderbilt University, Nashville 4, Tenn.
**Kneip,** George D., Jr., RFD 4, Chagrin Falls, Ohio.
**Kozora,** Andrew J., 1815 Lowris St., Pittsburgh, Pa.
**Larguier,** Rev. E. H., Spring Hill College, Mobile County, Ala.
**Lathrop,** Arthur L., 600 S. Gary Pl., Tulsa, Okla.
**Lawson,** Kent D., 1519 Sage Ave., Troy, N. Y.
**Leach,** Herbert W., Box 174, Tarleton Station, Tex.
**Leonelli,** Renato Edmund, Rhode Island College of Education, Providence, R. I.
**Lindblom,** Andrew Charles, 101 Hoyt St., Laramie, Wyo.
**Loveridge,** Lawrence E., 120 Ridge Rd., Rutherford, N. J.
**McNutt,** Wayne Eugene (*J*), 4415 N. Wayne Court, Ft. Worth, Tex.
**Meadows,** S. R., Idaho State College, Pocatello, Idaho.
**Mennen,** Sulamith B., University of Chattanooga, Chattanooga, Tenn.
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**Moore,** Donald Clark, 1513 6 St., Berkeley 2, Calif.
**Morgan,** Ira Lon (*J*), 444 Burnet St., Ft. Worth, Tex.
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**Neeland,** Gerald K., 845 Wealthy St., SE, Grand Rapids, Mich.
**Neher,** H. V., California Institute of Technology, Pasadena, Calif.
**O'Connell,** Walter D., University of Southern California, Los Angeles, Calif.
**Ostergaard,** Soren K., Buena Vista College, Storm Lake, Iowa.
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**Pezzuto,** Albert M., St. Ambrose College, Davenport, Iowa.
**Pierce,** Donald Frederick, Lake Forest College, Lake Forest, Ill.
**Plass,** Gilbert N., Johns Hopkins University, Baltimore 18, Md.
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**Roth,** Raymond E., University of Dayton, Dayton, Ohio.
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**Seale,** Robert L., 3405½ Amherst, Houston 5, Tex.
**Scheerer,** Carl W., 711 Market St. Lewisburg, Pa.
**Smith,** Francis J., 1312 Pond St., Bristol, Pa.
**Sopp,** Theodore Ernst, 5156 Pulaski Ave., Philadelphia 44, Pa.
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**Stavseth,** Richard M. (*J*), 3134 NE Weidler St., Portland 12, Ore.
**Stauffer,** A. J., Box 11, West Leesport, Pa.
**Stearns,** Jackie Illma, George Washington University, Washington, D. C.
**Stimson,** Sister Miriam Michel, Siena Heights College, Adrian, Mich.
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**Willard,** Wayne, Rochester Junior College, Rochester, Minn.
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# Proceedings of the American Association of Physics Teachers

## The Chicago Meeting, December 29-31, 1947

THE seventeenth annual meeting of the American Association of Physics Teachers was held at the University of Chicago on December 29–31, 1947. PROFESSOR BUCHTA was chairman of the program committee for the meeting.

A joint dinner with the American Physical Society was held on Tuesday evening, December 30, in the International House Theater, University of Chicago. After the dinner, PAUL E. KLOPSTEG, of Northwestern Technological Institute, spoke on the subject, "Bows and Arrows."

### Invited Papers and Reports

*Committee Reports*

**Employment of physicists at the B. A. level.** M. N. STATES, *Central Scientific Company.*

**Letter symbols.** H. K. HUGHES, *Socony-Vacuum Laboratories.*

**Teaching of physics and the physical sciences in the secondary schools.** L. W. TAYLOR, *Oberlin College.*

*Papers and Special Events*

**Utilization of physicists by the Army in World War II.** MARSH W. WHITE, *Pennsylvania State College.*

**Presentation of the Oersted Medal of the American Association of Physics Teachers to W. H. Barber.** R. C. GIBBS, *Chairman of the Committee on Awards,* and PAUL KIRKPATRICK, *President of the Association.*

**Forty years of physics at Ripon College.** W. H. BARBER, *Ripon College.*

**New frontiers—sixth Richtmyer Memorial Lecture of the American Association of Physics Teachers.** HOMER L. DODGE, *Norwich University.*

*Joint Session with the American Physical Society and Section B, American Association for the Advancement of Science*

**The physics of radar.** LEE A. DUBRIDGE, *California Institute of Technology.*

*Joint Symposium with the American Physical Society: Microwaves and Their Uses*

**Absorption of microwaves by hydrogen atoms.** WILLIS E. LAMB, JR., *Columbia University.*

**Applications of microwaves in molecular spectra.** WALTER GORDY, *Duke University.*

**Production and detection of microwaves.** OTTO H. SCHMITT, *University of Minnesota.*

**Microwaves as teaching aids.** C. L. ANDREWS, *New York State College for Teachers.*

### Contributed Papers, with Abstracts

One session was devoted to the following contributed papers:

**1. Demonstration of magnetic field inside a current-carrying conductor.** MARIO IONA, JR., *University of Denver,* AND JOHN P. KARBLER, *University of Chicago.*—Many demonstration experiments are in use for the analysis of the magnetic field of a current-carrying conductor. Usually only the field outside of the conductor is investigated. One of the frequently described experiments can well be adapted, however, to the investigation of the magnetic field inside of the conductor—an electrolyte. Magnets are mounted radially inside a straight cylindrical electrolytic conductor in such a way that they can turn about the axis of the cylinder. The resultant torque on the magnets is determined by the magnetic field strength at the poles and their distance from the axis. Using concentric electrolytic conductors, one can demonstrate that the field is not affected by currents outside but only by currents closer to the axis than the poles. This experiment, performed in connection with the one described by Maxwell using the magnets outside of the conductor where there is no resultant torque, clearly demonstrates the different dependence of the field strength on the distance from the axis of the conductor inside and outside.

**2. The teaching of heat and thermodynamics in elementary courses.** ERIC M. ROGERS, *Princeton University.*—A plea was made for a change of balance of topics in heat. A short account of thermodynamics deserves a place, even in elementary courses, as an example of an important scientific method; but it should be carried through at least one example of its application (such as the derivation of the Clausius-Clapeyron equation). A way of giving such an account was described.

**3. Wave theory in Cartoons.** ROBERT S. SHAW, *College of the City of New York.*—This was one of a series of exhibits which began before the Association in 1943. The present material is related to a relatively abstract subject, namely, wave theory.

**4. Discussion of thermocouples in textbooks.** G. K. SCHOEPFLE, *Kent State University.*—In the discussion of thermocouples in many textbooks there appears a parabola to illustrate the change in the electromotive force with

temperature for a copper-iron thermocouple. The student retains a visual memory of this parabola and expects that similar curves are obtained for any pair of metals, with the neutral point falling at a temperature obtained easily in the laboratory. This is not the case for common or commercial thermocouples, and a little thought will convince the student that the vertex of the parabola should not lie within the temperature range employed. An analysis was made of thermocouples as discussed in introductory textbooks.

**5. Factors to be considered in evaluating student results in the determination of the coefficient of friction.** SISTER MARY GRACE WARING, *Marymount College*.—A record of data obtained by different groups of students who determined the coefficient of friction from the same apparatus and using the same procedure showed that the results were varied. A study was made to ascertain the cause of discrepancies; it was limited to the determination of sliding friction on a horizontal surface. (i) A polished-oak friction block was moved over a finely polished hardwood surface by means of weights attached to a cord running over a pulley to the block. When the block was started from a given position in line with the pulley, its path was such that altogether different results were obtained than when the block was slightly out of alinement. When the string passing over the pulley was carefully leveled, and the block started from the alined position, the main cause of discrepancies was differences in the humidity on different days. (ii) A friction box having two leather strips glued to the bottom was moved over a board with a smooth, sandpapered surface. Discrepancies larger than those in (i) were observed. Results could be duplicated when the box was lifted to bring it back to the starting point, whereas another set of results appeared when the box was dragged back over the surface. Different conditions of humidity produced corresponding differences in the forces of friction between the two surfaces.

**6. Remarks on Melde's experiment.** E. W. CHENEY AND PAUL F. BARTUNEK, *Lehigh University*.—In performing Melde's experiment, using a telegraph sounder operated from a 60-c/sec line to excite the string, some of the beginning students observed the interesting result that the string seemed to be excited with two different frequencies. If a plot is made of the square of the number of vibrating segments *versus* the reciprocal of the stretching force, the slope of the resulting straight line is a measure of the square of the frequency. The fact that all of the points do not lie on a straight line need not worry the students. The points can be plotted as they come; then two or more straight lines result, one for each of the exciting frequencies and combination frequencies, if these appear. In most cases the observed frequencies were 60 and 120 c/sec. In one or two cases 180 c/sec was observed.

**7. Transverse wave apparatus.** F. E. CHRISTENSEN, *University of Minnesota*.—It is recognized that the problem of making an impulse given to a rope remain stationary with respect to an observer is difficult and, at best, can only be approximately solved. The assumption made in one derivation of the formula $v=(T/\rho)^{\frac{1}{2}}$ for the velocity of a transverse wave along a flexible string is that the string moves in a direction opposite to and with a velocity equal to that of the wave form. A continuous, loaded, rubber tube passing over two pulleys about 8 ft apart can be given a velocity very nearly equal to that of the wave. The apparent wave velocity is equal to the difference between the belt speed and the actual velocity of the wave along the rope. This apparent wave velocity is zero if the tube and wave velocities are equal. It can also be shown that waves of different forms travel with the same velocity.

**8. A small oxygen liquefier.** LESTER I. BOCKSTAHLER, *Northwestern University*.—Pure oxygen gas at a pressure of 2 lb/in.$^2$ is liquefied by passing it over a vacuum-jacketed tube containing liquid air. The liquid oxygen is received in an ordinary Dewar flask. This forms a very convenient way for obtaining modest amounts of pure liquid oxygen, the temperature of which can be used as a primary point in calibrating instruments for low-temperature measurements. The apparatus consists of three coaxial Pyrex tubes placed one inside the other. The innermost vessel is formed like a large test tube and contains liquid air. This is enclosed on the sides and bottom by a somewhat larger tube. The oxygen fills the annular space between the two tubes. These are surrounded by a third tube which provides a vacuum jacket. The oxygen gas is admitted through a small side arm at the top of the second tube, and the liquid trickles out at the bottom through a spiral sealed through the vacuum shield. With an assembly having over-all dimensions of $60\times 6$ cm the yield is 1 lit/hr of liquid for an expenditure of about 5 lit of liquid air.

**9. A new sign convention for geometric optics.** G. T. PELSOR, *The American University*.—The formula relating image and object distances for reflection or refraction at spherical surfaces can be written in a symmetric and easily remembered form if one agrees to use the absolute value of change in index of refraction, $|\Delta\mu|$, and to use the following sign conventions. (*a*) Radius of curvature $R$ is positive if an incident ray parallel to the axis is bent toward the axis, negative if it is bent away from the axis; that is, $R$ is positive for converging and negative for diverging surfaces. The same convention is followed for the focal length $f$ of a single surface or of a lens. (*b*) The object or image distance $D$ is positive if the object or image is real, negative if it is virtual. These conventions result in the formula

$$(\mu_0/D_0)-(\mu_n/D_n)=\Sigma(|\Delta\mu|/R)=\mu_0/f_0=\mu_n/f_n,$$

where the sum includes a term for each of $n$ surfaces separated by distances that are small compared with $D_0$ and $D_n$. For reflecting surfaces, $\Delta\mu$ is replaced by 2. It will be noted that no negative signs appear in the formula. All numerators are positive, so that each term follows the sign of its denominator as determined by the foregoing conventions.

**10. A system of electrical connections for experimental work.** Benjamin L. Snavely, *Naval Ordnance Laboratory.*—A convenient, versatile, inexpensive scheme for connecting small electrical equipment employs flexible leads with interchangeable end fittings by which connections can be made to any type of terminal or binding post. Leads are furnished with small female spring connectors designed to take a plug $\frac{3}{32}$ in. in diameter and $\frac{5}{16}$ in. long. The barrel-type contact found on some types of radio-tube socket may be used for this purpose. These are protected by short lengths of heavy plastic tubing so that the leads themselves have no exposed terminals. Plugs made from $\frac{3}{32}$-in. brass rod form a part of all fittings used with the leads. Breadboard circuits, generally impractical to assemble without solder, are built up on a steel chassis holding four vacuum tubes in an inverted position. The chassis has a vertical section on which potentiometers, switches and similar parts are mounted and a shelf-like base to hold heavier components. The arrangement is compact and easily shielded. Bus bars, specially shaped to facilitate soldering, and numerous tie points are provided. External connections to the breadboard are made with flexible leads that are attached to pins on the ends of the bus bars and to pins on the tie points.

**11. Production of air bubbles in water by a hot wire.** C. H. Tindal and D. C. Whitmarsh, *Ordnance Research Laboratory, The Pennsylvania State College.*—For demonstrating their effects—for example, on the transmission of energy through water—air bubbles may be produced in aerated water by an electrically heated wire. The rate of bubble production may be controlled by regulating the power supplied to the wire. For a volumetric rather than a planar distribution of bubbles the wire can be zigzagged or a grid or a screen can replace the wire. Hot-wire technics and apparatus were discussed. Simple equipment for insuring saturated aeration was described as well as that for determining the rate of bubble formation.

**12. Small bubble photography by a projection technic.** D. C. Whitmarsh and C. H. Tindal, *Ordnance Research Laboratory, The Pennsylvania State College.*—A method for measuring the size of small bubbles produced in water involves a combination projection and photographic technic wherein the image of the bubbles is focused on a focal-plane camera shutter behind which is placed a photographic film. The shutter is used to expose the film when the bubbles to be recorded are in the field. This projection method permits enlargement of the bubbles onto the film negative, which can be further enlarged for measurement purposes. The technic is applicable to recording or measuring objects capable of being projected, whether moving or stationary in the field of the projector.

**13. Simple apparatus for mapping electrical fields.** C. J. Overbeck, *Northwestern University.*—The dry type (Cenco) electric field mapping apparatus has distinct advantages over the older electrolyte tray method. A modification of the dry type was described, in which "resistance paper" and a U-shaped exploring probe are used. The paper and the data record sheet occupy opposite faces of a plywood or pressboard panel. The probe then locates comparable positions on both sheets, and the equipotential points may thus be recorded directly on the graph or other record paper. The potential terminals may be attached to the resistance paper by Scotch tape if desired. A potential difference of 6 v on the potential terminals and a Weston 440 galvanometer on the exploring probe will locate the equipotential points to within 1 mm.

**14. Conductivity of dilute water solutions near the critical temperature.** A. C. Swinnerton and G. E. Owen, *Antioch College.*—Attempts to measure the conductivity of water and solutions of low concentration through the critical temperature were described. The methods employed include tubular stainless-steel bombs with electrodes introduced through insulating connectors. Various frequencies from 60 to 10,000 c/sec have been applied. The conductivity is measured with an impedance bridge. The results show increasing conductivity with rising temperature in the liquid state until the critical region is approached; a much lower order of conductivity was found for the vapor phase, but it also increased with temperature. The two curves approach each other in the critical region, and appear to join in such a way as to indicate a transitional range of several degrees. The investigations are being made as part of a research contract with the Signal Corps Engineering Laboratories which calls for experimentation in the artificial crystallization of quartz of a size useful for piezoelectric purposes.

**15. Appropriate components in general physics laboratory experiments.** Louis R. Weber, *Colorado A. & M. College.*—Although accuracy is not the chief goal of the elementary laboratory, the choice of proper components in any experiment can contribute much to a reasonably accurate result. The student has the satisfaction of work well done and probably will remember the experiment and the principles longer. Some experimental set-ups are as absurd as the arrangement one might use to measure 100 ml of water by finding the height of water in a bathtub before and after the introduction of the water that was to be measured. In any apparatus, there are certain inherent errors and thus the equipment is more accurate in certain ranges. Students frequently get the idea that a simple wire form Wheatstone bridge is useful for measuring *any* range of resistance values. Because a set of resistors from some scientific company may include those of a fraction of an ohm, students are required to measure these; the errors introduced by the resistance of the connections and various parts of the bridge all but smother the resistance the student wishes to find. In an experimental set-up, we should choose our components to bring out desirable principles despite instrumental errors generally present. Other suggestions were given.

## Attendance

The following persons registered at the annual meeting:

Mildred Allen, Mount Holyoke College; K. L. Andrew, Friends University; C. L. Andrews, New York State College for Teachers; W. Azbell, Bradley University; R. H. Bacon, Bradley University; J. F. Baird, Bremen High School (Bremen, Ind.); W. M. Baker, University of Detroit; I. A. Balinkin, University of Cincinnati; W. H. Barber Ripon College; P. F. Bartunek, Lehigh University; C. J. Baumgaertner, College of St. Thomas; Louise G. Belai, Our Lady of Cincinnati College; P. Bender, Goshen College; C. E. Bennett, University of Maine; D. M. Bennett, University of Louisville; E. W. Beth, AMC Electronic Research Laboratories; W. H. Billhartz, Western College; O. Blackwood, University of Pittsburgh; B. F. Boardman, AEC, Oak Ridge; L. E. Bockstahler, Northwestern University; P. E. Boucher, Colorado College; J. M. Bradford, Beloit College; M. L. Braun, Catawba College; G. P. Brewington, Lawrence Institute of Technology; Brother Bruno, Cathedral High School (Indianapolis, Ind.); J. W. Buchta, University of Minnesota; E. L. Bussell, Chicago Technical College; A. B. Cardwell, Kansas State College; H. E. Carr, U. S. Naval Academy; T. Century, Senn High School; F. E. Christensen, University of Minnesota; C. A. Cinnamon, University of Wyoming; C. C. Clark, New York University; F. M. Clark, Wayne University; Velda Clurhe, Compton's Encyclopedia; W. W. Colvert, Illinois Institute of Technology; D. Compton, Wabash College; W. C. Connolly, U. S. Naval Academy; Sister Maria Consolata, St. Mary's College (Notre Dame, Ind.); S. W. Cram, Kansas State Teachers College; J. Daintz, NRC, Canada; J. B. Davis, Lower Merion Senior High School (Ardmore, Pa.); Walter M. DeCew, Nucleonics; R. J. Derenthal, College of St. Thomas; B. H. Dickinson, Michigan State College; E. H. Dixon, University of Georgia; H. L. Dodge, Norwich University; Sr. M. Dulciosa, St. Joseph High School; W. E. Dyer, Lewis College of Science and Technology; D. J. Eaton, Northern Illinois State Teachers College; D. A. Edwards, Lincoln University; R. L. Edwards, Miami University; J. D. Elder, Wabash College; O. C. Estes, University of Illinois; P. E. Fossum, St. Olaf College; O. G. Fryer, Drury College; H. Q. Fuller, Missouri School of Mines and Metallurgy; M. Garbuny, Westinghouse Electric Co.; G. M. Giddings, General Electric Research Laboratory; H. Gilbarg, Purdue University; W. Gordy, Duke University; W. L. Groenier, Herzl Junior College; H. E. Hammond, University of Missouri; R. Hanau, University of Kentucky; R. T. Harling, St. Lawrence University; H. H. Hartzler, Goshen College; J. C. Hendricks, Franklin College; R. L. Henry, Carleton College; M. E. High, University of Kansas City; C. W. Hill, Schurz High School (Chicago, Ill.); W. A. Hilton, William Jewell College; J. W. Hornbeck, Kalamazoo College; F. F. Householder, University of Akron; G. Horinchi, University of Chicago; F. T. Howard, University of Denver; R. H. Howe, Denison University; E. I. Howell, Belhaven College; J. H. Howey, Georgia School of Technology; H. K. Hughes, Socony-Vacuum Laboratories; K. Hujer, University of Chattanooga; E. Hutchisson, Case Institute of Technology; H. M. James, Purdue University; E. N. Jensen, Iowa State College; H. C. Jensen, Lake Forest College; J. C. Jensen, Nebraska Wesleyan University; A. M. Johnson, University of Illinois; P. G. Johnson, U. S. Office of Education; Sister Maria Jose, Immaculata College; Rosemary Kadolph, St. Xavier College; C. R. Kelly, Loyola University; Mother M. Kernaghan, Maryville College; A. A. Kildare, Lincoln University; W. H. Kinsey, University of Connecticut; P. Kirkpatrick, Stanford University; P. E. Klopsteg, Northwestern University; E. Kotcher, Air Force Institute of Technology; W. E. Lamb, Jr., Columbia University; H. M. Lashier, Emmanuel Missionary College; T. W. Lashof, Reed College; F. I. Leib, Michigan State Normal College; R. B. Lindsay, Brown University; G. A. Litchfield, Jr., Lake Forest College; E. M. Little, Naval Ordnance Laboratory; F. W. Loomis, University of Illinois; A. L. Lutz, Wittenberg College; K. V. Manning, Pennsylvania State College; C. L. Mason, Rose Polytechnic Institute; G. H. Mason, Eureka College; J. W. Marshall, United States Rubber Co.; W. W. McCormick, University of Michigan; R. O. McIntosh, Westinghouse Research Laboratories; W. J. McGonogle, Western Michigan College; Charlotte V. Meeting, McGraw-Hill Book Co.; J. W. Michener, Carnegie Institute of Technology; W. H. Michener, Carnegie Institute of Technology; H. A. Miley, Air Materiel Command, Electronic Research Laboratories; C. W. Miller, Michigan State College; W. B. Miner, Bradley University; J. Moelk, Wright Junior College; J. Mokre, Barat College; F. V. Monaghan, Michigan State College; C. L. Moore, Doane College; C. S. Morris, Manchester College; W. W. Mutch, Knox College; H. E. Newhard, Findlay College; W. Noll, Berea College; N. Li, St. Louis University; W. V. Norris, University of Oregon; R. G. Nuckolls, Illinois Institute of Technology; Sister Jeanette Obrist, Mt. St. Scholastica College; Sister Michael Edward O'Byrne, Incarnate Word College; F. B. Oleson, University of Maine; C. J. Overbeck, Northwestern University; G. E. Owen, Antioch College; Mother M. L. Padberg, Maryville College; R. R. Palmer, Beloit College; L. A. Pardue, University of Kentucky; R. F. Paton, University of Illinois; G. T. Pelsor, American University; Mary W. Peters, University of Tennessee; W. E. Peterson, Herzl Junior College; E. R. Phelps, Wayne University; D. F. Pierce, Lake Forest College; E. G. Pigg, North Georgia College; E. R. Pinkston, U. S. Naval Academy; J. Platt, University of Chicago; J. G. Potter, A. & M. College of Texas; W. E. Pullin, State A. & M. College, South Carolina; O. L. Railsback, Eastern State College, Illinois; T. S. Renzema, Purdue University; C. O. Riggs, Waynesburg College; J. A. Rinker, Eureka College; E. Ritchie, Centre College; Eric Rogers, University of Alabama; V. Rojansky, Union College; P. Rood, Western Michigan College; W. R. Rusk, University of Tennessee; Sister M. St. Bernard, Immaculata College; T. Sando, Wabash College; R. A. Sawyer, University of Michigan; R. R. Schiff, Westinghouse Research Laboratories; H. K. Schilling, Pennsylvania State College; C. C. Schmidt, Texas Technological College; G. K. Schoepfle, Kent State University; F. W. Sears, Massachusetts Institute of Technology; P. C. Sharrah, University of Arkansas; R. S. Shaw, City College, New York; L. S. Shweitz, Ball State Teachers College; R. Shwietz, University of Michigan; W. Shockley, Bell Telephone Laboratories; A. F. Silkett, University of Illinois, Navy Pier Branch; C. R. Smith, Aurora College; H. L. Smith, Michigan State Normal College; L. E. Smith, Denison University; O. H. Smith, DePauw University; Z. L. Smith, University of Chicago; J. R. Smithson, U. S. Naval Academy; B. L. Snavely, Naval Ordnance Laboratory; R. W. Snyder, University of Michigan; D. L. Solteau, University of Redlands; A. D. Sprague, Depauw University; M. N. States, Central Scientific Co.; E. L. Steele, Cornell University; R. J. Stephenson, College of Wooster; G. W. Stewart, University of Iowa; R. M. Sutton, Haverford College; K. R. Symon, Wayne University; P. A. Tapley, Chicago High School; C. D. Thomas, West Virginia University; C. H. Tindal, Ordnance Research Laboratory; J. Todd, G. & C. Merriam Co.; P. S. Townsend, University of Dayton; J. S. Urban, Centenary College; G. D. Van Dyke, Earlham College; J. J. Vasa, De Paul University; A. L. Vaughan, University of Minnesota; F. Verbrugge, Carleton College; H. R. Voorhees, University of Chicago; R. C. Waddell, University of Illinois; C. C. Walker, State College, Orangeburg, S. C.; C. N. Wall, University of Minnesota; Sister Mary Grace Waring, Marymount College; K. Watanabe, Wabash College; T. F. Watson, University of Wichita; L. R. Weber, Colorado A. & M. College; M. W. White, Pennsylvania State College; D. C. Whitmarsh, Pennsylvania State College; L. W. Whitney, Southwest Missouri State College; G. C. Wick, Notre Dame University; W. A. Wildhack, National Bureau of Standards; W. Willard, Rochester Junior College; G. H. Winslow, Argonne National Laboratory; R. M. Woods, Westminster College; C. O. Woodward, Central State College (Okla.); F. T. Worrell, R. P. I.; C. M. Yager, North Georgia College; Pearl I. Young, Pennsylvania State Center, Pottsville; M. W. Zemansky, City College, New York.

## Report of the Secretary

The Executive Committee held its annual meeting on December 29, 1947. President Paul Kirkpatrick presided.

The following persons were present: *Mildred Allen, *D. M. Bennett, O. H. Blackwood, *P. E. Boucher, *J. W. Buchta, H. L. Dodge, J. D. Elder, *R. C. Gibbs, *P. Kirkpatrick, *P. E. Klopsteg, *W. V. Norris, T. H. Osgood, *C. J. Overbeck, J. G. Potter, *O. L. Railsback, *C. O. Riggs, *H. K. Schilling, *O. H. Smith, *D. L. Soltau, M. N. States, *J. D. Stranathan, R. M. Sutton, *L. W. Taylor, M. H. Trytten, *H. R. Voorhees, M. W. White, M. W. Zemansky. Asterisks indicate the names of voting members or proxies. The others were present by invitation.

*The Journal.*—Upon recommendation of the editor, R. H. Bacon, E. L. Hill, and A. T. Jones were appointed associate editors for the three-year period 1948–1950.

The Executive Committee during the past two months, following the resignation of Duane Roller as Editor, made arrangements to secure the services of T. H. Osgood to fill this vacancy. In order to complete the appointment, they accepted his resignation as the newly elected Vice President and appointed him Editor of the *American Journal of Physics* for a three-year term.

*Business with American Institute of Physics.*—Our dues for 1948 to the American Institute of Physics, as a supporting member society, have been set at 15 percent of our 1947 dues income. We have approved an increase in the per-page publication charge from $3.00 to $4.00. The Institute has installed modern addressing machines and will take over the membership mailing of the Association.

We have been informed that the first copy of the new publication, *Physics Today*, will be sent to Association members within a few months. The Institute has requested assistance in two projects: (1) compilation of information on special fields emphasized in the departments of physics of the colleges and universities of this country, (2) the production of a booklet on the subject, "Careers in physics."

P. E. Klopsteg was recommended as our new representative on the Governing Board of the Institute. Our representatives and their terms of office are: R. C. Gibbs, 1946–49; Marsh White, 1947–50; P. E. Klopsteg, 1948–51.

*Reports of officers and committees.*—The policy committee and the journal and budget committees submitted their plans and proposed budget for the year. To carry out the projects, which include issuance of a new membership directory, extension of the work of our committees in cooperation with other science organizations, extension of our membership campaign, and so forth, we will operate, during 1948, on a budget slightly larger than our anticipated income. A study of the Treasurer's report, which appears elsewhere in this issue, should allay any immediate fear of this procedure.

Our membership committee, under the chairmanship of Marsh White, has made a record increase. There is every indication that the membership will soon reach the 2000 mark.

Reports were given by our representatives working in cooperation with other organizations. These include: American Council on Education; American Association for the Advancement of Science; National Science Teachers Association; Inter-Society Committee on a National Science Foundation; American Standards Association, and American Society for Engineering Education. In addition, the Chairmen of Association committees on subject matter presented reports. Our active committees include: teaching of physics and the physical sciences in the secondary schools; physics in premedical and medical training; letter symbols and abbreviations; terminology; Coulomb's law; optical projection.

Three new committees dealing with Association procedure have been set up. These committees are: emeritus and life memberships; election procedures; and a committee to study our constitution, by-laws and minutes in the light of our present growth and needs.

*Regional sections.*—The Wisconsin Association of Physics Teachers presented a petition to be recognized as the Wisconsin Section of the Association. This recognition was given, and thus the group becomes our eleventh regional section.

*Future meetings.*—The ASEE has again invited the Association to hold a cooperative summer meeting, and we have accepted. The next meeting will be held June 14–18 in Austin, Texas.

*Annual business meeting.*—The seventeenth annual business meeting was held on December 30, 1947, at 9:30 A.M. By vote in the annual election and by action of the Executive Committee, the outcome of the election was as follows:

*President:* J. W. Buchta.
*Vice President:* H. K. Schilling.
*Executive Committee:* R. F. Paton; E. M. Rogers.

To facilitate the collection of membership dues, it was voted to change the annual date of collection from February 1 to January 1.

In recognition of Duane Roller's fine service to the Association, the Executive Committee directed that the following resolution be submitted at the annual Business Meeting:

> During the entire period in which the Association has supported a publication for its membership, it has been exceedingly fortunate in having Dr. Duane Roller as its editor. Under his able management and with his untiring efforts and exceptional creative talent, the *American Journal of Physics* has developed into a periodical of prestige and distinction in its field; and its influence upon the growth of the Association to its present stature and significance has been incalculable. During the years of his editorship, the Association has also had the benefit of Mrs. Roller's able assistance in the editorial office. In view of Doctor Roller's retirement as editor, the American Association of Physics Teachers, assembled in annual meeting, takes the occasion to express to him its recognition and deep appreciation of his devoted and effective services through the years, and wishes for him and Mrs. Roller the realization of some of the

leisure which they have earned, and the pleasure and contentment which the relief from the pressing duties of the editorship may afford them.

The members present at the annual meeting responded by a rising vote of approval and appreciation.

C. J. OVERBECK
*Secretary*

## Annual Report of the Treasurer

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| *Balance brought forward from December 15, 1946* | | $ 7633.61 |
| CASH RECEIVED | | |
| Dues received for 1947 | $7714.83 | |
| Dues received for 1946 | 50.00 | |
| Dues received for 1948 | 1080.00 | |
| Royalties, *Demonstration experiments in physics* | 261.31 | |
| Constituent Membership in ACE, paid by American Institute of Physics | 100.00 | |
| Donation | 100.00 | |
| Matured U. S. Treasury Bond | 3000.00 | |
| Bond coupons | 135.00 | |
| *Total deposited, 12/15/46 to 12/15/47* | | 12,441.14 |
| Total cash available | | $20,074.75 |
| DISBURSEMENTS | | |
| Postage and supplies, editor's office | 199.55 | |
| Postage and supplies, treasurer's office | 203.10 | |
| Secretary's office expenses | 256.15 | |
| Stenographer, editor's office | 620.40 | |
| Salary of assistant editor | 600.00 | |
| Travel expenses, editor and assistant editor | 273.10 | |
| Expenses of membership committee | 271.18 | |
| Expenses of AAAS Cooperative committee | 162.57 | |
| Printing | 659.05 | |
| Payments to American Institute of Physics | 3122.50 | |
| Constituent Membership in ACE | 100.00 | |
| Oersted certificates (10) | 100.00 | |
| Purchase of U. S. Treasury Bonds | 8257.19 | |
| Miscellaneous travel expenses | 103.86 | |
| Bank expenses, discount on checks, purchase of bonds, etc. | 14.46 | |
| Miscellaneous | 2.90 | |
| *Total disbursed* | | 14,946.01 |
| Balance on hand December 15, 1947* | | $ 5128.74 |

PAUL E. KLOPSTEG, *Treasurer*

I have audited the books of account and records of Dr. Paul E. Klopsteg, Treasurer of the American Association of Physics Teachers, for the year ended December 15, 1947, and hereby certify that the foregoing statement of receipts and disbursements correctly reflects the information contained in the books of account. Receipts during the year were satisfactorily reconciled with deposits as shown on the bank statements, and all disbursements have been satisfactorily supported by vouchers or other documentary evidence. U. S. Government Bonds and Notes of $10,000 par value are held in safekeeping by the State Bank and Trust Company of Evanston, Illinois, and a certificate was obtained from the custodian as of December 15, 1947.

WILLIAM J. LUBY
*Certified Public Accountant*

Evanston, Illinois,
December 22, 1947.

* Approximately $1000 is still due the American Institute of Physics for publication of the journal in 1947. In addition to the balance shown, the Association holds U. S. Government Treasury Bonds and Notes of par value $10,000.

*Spiritual force, history clearly teaches, has been the greatest power in the development of men and history. Yet we have merely been playing with it and never seriously studied it, as we have the social forces. Some day people will learn that material things do not bring happiness and are of little use in making men and women creative and forceful. . . . When this day comes, the world will advance more in one generation than it has in the past four generations.*—CHARLES STEINMETZ.